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**ROBUSTNESS IMPROVEMENT OF ACTIVELY  
CONTROLLED STRUCTURES THROUGH  
INTEGRATED STRUCTURAL/CONTROL DESIGN**



S. S. Rao and T. S. Pan

School of Mechanical Engineering  
Purdue University  
West Lafayette IN 47907

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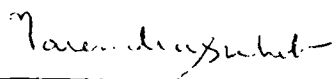


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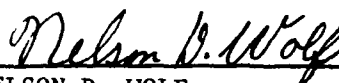
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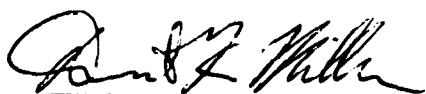


NARENDRA S. KHOT  
Project Engineer  
Design & Analysis Methods Group



NELSON D. WOLF  
Technical Manager  
Design & Analysis Methods Group  
Analysis & Optimization Branch

FOR THE COMMANDER



DAVID K. MILLER, Lt Col, USAF  
Chief, Analysis & Optimization Branch  
Structures Division  
Flight Dynamics Directorate

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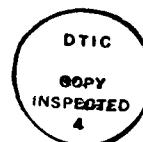
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## CHAPTER 1

### INTRODUCTION

With increasing space activity, the use of light and flexible structures is becoming important to reduce the high cost of lifting mass to orbit. The most important types of structures used in space include large solar arrays and antennas, deployable satellites and structures, precision optical systems, and space stations. Because of their flexibility, low and closely-spaced resonant frequencies, and low damping coefficients, advanced structural and control techniques are required to achieve the stringent pointing, displacement and shape accuracy requirements for space structures. It has been found that passive control, combined with active control, will be necessary for large flexible structures. The interrelationship among the various disciplines affecting the analysis and design of large flexible structures is presented in Figure 1.1. This diagram shows that the aspects of modeling of the structure, control system design and location selection of control devices are very closely related and play central roles in the flexible structural control problem. The problem of control system design of large flexible structures can be broadly divided into two major parts - modeling and controller design. Flexible structures are usually, by nature, distributed parameter (continuous) systems whose dynamics can be modeled by partial differential equations. A popular method to generate a finite-degree-of-freedom model for approximating a flexible structure, which has infinite degrees of freedom, is the finite element method (FEM). Because of the large number of degrees of freedom in the configuration space, the finite element models of large flexible structures are of extremely high order, typically involving thousands of variables. It is impractical to design a control system by using the full order finite element model. The model reduction procedure is used to reduce the high order finite element model to a feasible lower order model for controller design.

The model reduction procedures might introduce the observer and control spillover problems which are caused by the unmodeled modes being measured and excited by the controller. The instability caused in a simple beam because of spillover was demonstrated by Balas (1978). Although much effort has been expended in control design or prefilter design to filter out the signal from the unmodeled modes, the fundamental problem has not been totally solved. Several approaches are proposed to eliminate the effect of spillover by Longman (1979), Sesak and Likens (1979), Lin (1980), Meirovitch and Baruh (1981), Czajkowski and Preumont (1987), and Preumont (1988).

As stated, the problem of controller design is a major issue in the control of flexible structures. We can classify the controller design into two major types, namely, the passive control and the active control. The passive control can be used to change the natural frequencies and damping coefficients only by small amounts; also it might increase the stability margin and the robustness of the system. The active control can modify the closed-loop performance by changing the damping ratio and/or the stiffness to some extent. However, the

active control performance might be greatly degraded by the spillover effect of the residual modes or the errors and uncertainties present in the system model. The general aspects of structural control were discussed by Leipholz (1979) and Leipholz and Abdel-Rohman (1985).

To meet the stringent performance requirements of flexible space structures, several challenging problems should be overcome. The first challenge is to generate an appropriate reduced order model for representing an infinite dimensional distributed structural system. This is a discrete input space-continuous output problem. The second challenge is to select the number and locations of the sensors/actuators. This selection problem directly affects the stability of the closed-loop system (Balas 1978) with the controller designed on the reduced model. The third challenge deals with robust controller design which retains the closed-loop stability and performance requirements under some amount of uncertainty in system parameters or design variables.

In practice, the order of the finite element model involved in large flexible structures is so high that it is impossible to solve a control design problem of the full order system. Hence, the model reduction problem becomes very important. To describe the model reduction problem, consider the dynamics problem involved:

$$\dot{x} = A x + B u \quad (1.1)$$

$$y = C x \quad (1.2)$$

where  $x, u$ , and  $y$  are the state, control, and the output vectors, respectively,  $A, B$ , and  $C$  are the state, input, and the output matrices, and the dot indicates the time derivative. From this, a reduced order model ( $A_c, B_c, C_c$ ) with reduced order state vector  $x_c$  will be found so the error criteria, established by the designer, will be minimized.

$$\dot{x}_c = A_c x_c + B_c u \quad (1.3)$$

$$y = C_c x_c \quad (1.4)$$

A graphical representation of the reduced order model is shown in Figure 1.2.

In general, the model reduction methods can be divided into two main categories as: model reduction in frequency domain and model reduction in time domain. Several comparative studies in model and controller reduction were made by Decoster and Cauwenberghe (1976), Elrazaz and Sinha (1981), Hyland (1984), Sugimoto et al. (1985), Hyland and Bernstein (1985), Parry and Venkayya (1986), Anderson and Liu (1987), and Oliver (1987). Most of the existing approaches used in model reduction involve an optimization problem which finds a set of parameters for the reduced order model so that a stated error criterion is minimized. The model decomposition procedure is an alternative for the modeling of large flexible structures. The substructures technique, which has been used in the finite element method for solving large complex structural analysis problems, was extended

to the modeling of large flexible structures for control design by Young (1988) and Pan et al. (1989). As described in Pan et al. (1989), the problems of model reduction and the spillover problems are avoided by using the substructures technique.

Since 10 years ago, the literature related to the modeling and the control of flexible space structures has been greatly increasing. Several survey papers on the subject have been presented in the past by Canavin (1978), Blair (1978), Dahlgren and Gunter (1978), Gran and Rossi (1979), Seltzer (1979,1980), Balas (1982), Garibotti (1984), Nurre et al. (1984), and Santiago et al. (1984). The aspects of passive control, decentralized and hierarchical control, integrated structural and control design, and number and/or location selection of sensors/actuators are reviewed in the following sections.

### **1.1 Passive Control in Flexible Structures**

The passive control methods for vibration suppression have been studied and used extensively for a long time. The advantages of passive control are as follows:

1. It is simple and inherently stable.
2. No power-source is needed.
3. It can increase the stability margin of the overall system.
4. The passive damping augmentation might lead to reliable and robust systems with simplified active controls requirements.
5. The proper design of the passive control system can reduce the settling time of the transient response and the peak-overshoot.

Hughes (1980) and Zak (1988) investigated the effect of passive energy dissipation in large space structure (LSS) control design. In the passive control, addition of damping (damper) and stiffness (spring or tendon) is usually used. Passive damping can be added to flexible structures through a variety of approaches including the addition of discrete viscous dampers, tuned-mass dampers, impact/friction joints, and constrained layer materials. The applications of tuned-mass dampers in high rise buildings to control the vibration of structures by the single-input single-output concept were discussed by McNamara (1977), Wiesner (1979), and Petersen (1980). Juang (1984) presented a technique for formulating expressions of the optimal tuning law for an elastic system including a truss beam and a tip vibration absorber. A formula relating the beam parameters to the size of the tip absorber is provided for the optimal design of an absorber along with an evaluation of its performance for the single mode case. Sesak et al. (1986) investigated the optimal tuning of multiple tuned-mass dampers for the transient vibration damping of large space structures by using modern control theory and parameter optimization techniques. Trudell et al. (1980) addressed the subject of passive damping provided by the use of viscoelastic materials for large space structures. In the PACOSS (Passive and Active Control of Space Structures) program, a representative system (Representative System Article) was developed and analyzed by Morgenthaler and Gehling

(1986). The modal strain energy distribution approach was used to determine locations where passive damping treatments would be most effective in a structure. Gehling (1986) also presented the benefits of passive damping with regard to active control implementation and retargeting performance of a large space structure.

Damping synthesis for flexible space structures has been studied by Soni and Agrawal (1985), and Simonian (1986). Vegte and Hladun (1973) proposed an optimal passive beam vibration control design by using optimal control techniques. Prucz (1987) suggested that the passively damped joints using high damping viscoelastic materials, have the potential of being effective practical means of passive vibration control.

A review of the damping mechanisms in advanced fiber-reinforced composites was given by Adams (1986). Prucz and Fu (1988) presented the feasibility of controlling the propagation of mechanical vibration associated with elastic stress waves in fiber-reinforced composites by appropriate tailoring of material constituents and fiber orientations. Ashley (1984) presented a study of the size effects on various damping mechanisms and thermal dissipation induced by strain gradients during vibration of monolithic configurations. Chen and Wada (1988) studied the application of viscoelastic damping materials employed in the parallel load path.

The stiffness modification in passive control is usually used to either maintain the shape of a reflective surface (e.g. Herbert and Bachtell, 1986) or reduce the vibration amplitude by increasing the stiffness. Many passive tendon control applications can be found in the structures of tall buildings and long-span bridges to achieve higher strengths. Roorda (1975) used the active tendon control in tall structures. An experimental tendon control system for a flexible space structure was investigated by Murotsu, et al.(1988). Although the passive control has some advantages, it is usually coupled with active control, to obtain what is known as a semi-active or hybrid control, to achieve the stringent dynamic requirements of flexible structures.

## **1.2 Decentralized and Hierarchical Control**

The models of large scale mechanical and structural systems are usually well known but often very complex. The system, sometimes, is viewed as a set of decoupled subsystems (Siljak, 1979), and a local controller (decentralized control, Singh, 1981) is synthesized for each subsystem neglecting the coupling among the subsystems in the first step. To satisfy a specified global performance requirement, a global control (hierarchical control) may be introduced. For most large dynamic systems (such as LFS), they usually can be described as collections of  $N$  interconnected subsystems (or substructures). For the  $i^{\text{th}}$  subsystem, let  $x_i$  be an  $n_i$  dimensional state vector,  $u_i$  be an  $m_i$  dimensional control vector and  $z_i$  be an  $r_i$  dimensional vector of interactions which come in from the other subsystems. The hierarchical structure of a controlled structure is presented in Figure 1.3. The subsystem dynamics are assumed to be linear and can be represented by the following state space equations:



$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + C_i z_i(t) \quad (1.5)$$

$$x_i(0) = x_{i0} \quad (1.6)$$

It is assumed that the interaction vector  $z_i(t)$  is a linear combination of the states of the  $N$  subsystems:

$$z_i(t) = \sum_{\substack{j=1 \\ j \neq i}}^N L_{ij} x_j(t) \quad (1.7)$$

It is desired to choose the control input  $u_1, u_2, \dots, u_N$  such that the performance index  $J$  is minimized. The performance index for a linear time-invariant system can be defined as:

$$J = \sum_{i=1}^N \left[ \frac{1}{2} |x_i(t_f)|^2_{Q_i} + \frac{1}{2} \int_{t_0}^{t_f} \left( |x_i(t)|^2_{Q_i} + |u_i(t)|^2_{R_i} \right) dt \right] \quad (1.8)$$

where  $x_i$  and  $z_i$  satisfy Eqs.(1.5)-(1.7), and  $|x|^2_Q = x^T Q x$ , and  $Q_i$  and  $R_i$  are positive semidefinite and positive definite, respectively. Survey papers in decentralized/hierarchical control are provided by Sandell et al.(1978), and Lindner and Riechard (1986).

A heuristic approach for the control of serially connected dynamical systems with and without time delays between the subsystems was proposed by Singh and Coales (1975). The control strategy is determined by minimizing the cost function, which is defined as the summation of output error squares and input energy for each subsystem during the whole control process. Singh (1975) presented a two-level algorithm which can achieve the optimum decentralized control for large interconnected systems. A comparison of two hierarchical optimization techniques was presented by Singh and Hassan (1976). Singh et al. (1976) developed a decentralized computational procedure to find the optimal feedback gain matrices of high-order linear quadratic problems. The interaction prediction principle is used for the development of the closed-loop decentralized control. Siljak (1976) proposed a decomposition-aggregation method using both the passive and the active stabilization devices for a spinning flexible spacecraft stabilization problem. A decentralized optimal control for stabilization of stabilizable large-scale systems was derived by Ikeda et al. (1983). The optimal control in a decentralized scheme is achieved by selecting an appropriate performance index. It is shown that optimality of the control is preserved for a modified performance index under perturbations in interconnections so that the strength of coupling does not increase. McClamroch (1985) presented a simple form of hierarchical control in the vibration suppression of flexible structures including the dynamics of actuators. First, a decentralized controller for the flexible structure is developed, and then a compensation is provided for the member damper actuators to suppress the effects of actuator dynamics. The use of decentralized control structures for LSS was investigated by Medanic et al. (1987). The

frequency weighting and the projective control techniques are used to ensure that the control effect does not spillover into the unmodeled dynamics. Ozguner and Yurkovich (1987) proposed an approach for decentralized frequency shaping for incorporation of frequency-domain (bandwidth) constraints into the control design of a large flexible space structure with the dynamic model including actuator dynamics.

The decentralized/hierarchical control procedure for LFS addressed by Janschek and Surauer (1987), describes a multi-input multi-output design procedure based on a well known sequential design approach. Local (low level) control loops for baseline stabilization and high level control loops for overall performance are designed to satisfy individual criteria. Bernstein (1987) investigated the sequential design of decentralized controllers by using an optimal projection approach.

Ozguner and Yurkovich (1986) presented a control strategy involving a decentralized model reference adaptive approach (DMRAC) using a variable structure control. Experiments on NASA's flexible grid were performed by using DMRAC. The DMRAC method was also investigated by Lee et al. (1988). It is shown that this method can achieve either output regulation or output tracking with adequate convergence, provided the reference model inputs and their time derivatives are integrable, bounded, and approach zero as time goes to infinity.

Vukobratovic and Stokic (1984) presented an iterative procedure for the suboptimal synthesis of a robust decentralized control for a large-scale nonlinear mechanical system to a set of nominal trajectories which are prescribed together with a region of allowable parameter values. A decentralized control law for a class of nonlinear interconnected systems was proposed by Saberi (1988). This optimal decentralized control law is derived by defining an appropriate local performance index for each isolated subsystem and solving these local optimal control problems.

West-Vukovich et al. (1984) presented the decentralized robust servomechanism problem with constant disturbances/set points for LFS. It is shown that the spillover problem can be eliminated. Lindner (1985), and Davison and Gesing (1987) considered the control of a flexible spacecraft using a decentralized technique. The model order reduction in decentralized control was discussed by Ozguner and Lee (1983), and Yousuff et al. (1986). The robustness property of reduced order decentralized control designs was studied by Young and Siljak (1985). Additional studies on the decentralized control of large scale systems can be found in Aoki (1968), Singh and Hassan (1979), Siljak (1979), Singh (1981), Xinogalas et al. (1982), and Davison (1984).

### **1.3 Integrated Structural and Control Design**

In the conventional structural control area, the structural design and the control design are treated as two separate procedures. Although each design is optimal, based on the individual criterion, the combined system might not be optimal in the global sense. Hence, a great deal of research is currently in progress on developing methods for the simultaneous (or integrated) design of the structure and the control systems. The current methodology applied

to the integration of the optimal process for structures and controls is discussed by Wisshaar et al. (1986). Cooper et al. (1986A,1986B) described IMAT (Integrated Multidisciplinary Analysis Tool) for the integrated control/structure design.

The improvement of a control system's properties, such as stability, controllability and sensitivity, of a flexible structure by changing the structural parameters has been studied by Venkayya et al. (1985), Eastep et al. (1986), Haftka et al. (1984,1987), Khot et al. (1984,1986), Flotow (1986), Schmit (1987) and Rao et al. (1989). The optimal design of flexible space structures with constraints on dynamic properties (or response) were considered by Canfield et al. (1986), and Woo (1987). Some optimization techniques used in structural multiobjective design were discussed by Rao (1984,1986,1987). Yedavalli and Skelton (1983), and Yedavalli (1984) presented the determination of the critical parameters of LFS with uncertainty by using parameter sensitivity analysis methods. The sensitivity studies in controlled structural design were presented by Gilbert (1986), Manning et al. (1986), and Adamian and Gibson (1987).

The weight of the structure was minimized with constraints on the distribution of the eigenvalues and/or damping parameters of the closed-loop system by Khot et al. (1985). The structure/control system optimization problem was formulated by Khot et al. (1987) with constraints on the closed-loop eigenvalue distribution and the minimum Frobenious norm of the control gains. A unified algorithm for sequential (or simultaneous) design modifications of a closed-loop constant gain control system and the flexible structure to be controlled was presented by Junkins et al. (1984). Hale (1985) considered an ellipsoidal set-theoretical approach to the integrated structural/control synthesis for vibration regulation of flexible structures. This approach attempts to maximize the allowable magnitude of an unknown (but bounded) disturbance to the structure while explicitly satisfying specific input and output constraints. Both structural parameters and control gains are variables during the optimization process. A sequential quadratic programming technique was used to solve the optimal structural/control design problem by Tseng and Arora (1988). They solved the boundary value problem associated with the optimally controlled structure by first treating it as an initial value problem and then solving it by using the iterative method of nonlinear quadratic programming.

Belvin and Park (1988) presented a method for the optimization of the closed-loop structural system using only structural tailoring. The optimal linear quadratic regulator control theory, in conjunction with modal-space control, is used with the weighting matrices chosen, based on physical considerations. A disturbance model for the integrated structural and control design problem was proposed by Slater (1988). This approach uses the response to dynamic inputs and constraint limits to establish trade-offs between the control energy and the structural mass. Lim and Junkins (1987) presented a design algorithm with numerical applications using stability robustness measures. Three different cost functions, namely, the total mass, stability robustness and the eigenvalue sensitivity, have been optimized with respect to a unified set of design parameters which include structural and control parameters and actuator locations.

In the last few years, the simultaneous control/structural design problem has been treated as a multiobjective optimization problem. It can be formulated as:

$$\text{Min } \vec{f}(\vec{x}, \vec{u}) \quad (1.9)$$

$$\vec{x}, \vec{u}$$

subject to

$$\vec{g}(\vec{x}, \vec{u}) \leq 0$$

where  $\vec{f}$  is vector of objectives, and  $\vec{g}$ ,  $\vec{x}$ , and  $\vec{u}$  are constraint, structural design, and control input vectors, respectively.

Salama et al. (1986), and Miller and Shim (1986) considered the simultaneous minimization, in structural and control variables, of the sum of the structural weight and the infinite horizon linear regulator quadratic control cost. A weighted-sum method for solving the multiobjective optimization problem in the simultaneous structural and control design of flexible structures was proposed by Hale et al. (1985). A weighted total mass consisting of structural design parameters is added to the cost functional and the structural parameters are varied to find the minimum total cost, and the mass and stiffness distributions of the structure are determined as a part of the optimization problem. Onoda and Haftka (1987) presented a similar optimization approach to solve the simultaneous design problem. It is proposed to minimize the weighted structural mass and weighted input energy subject to the output norm constraints by finding the optimal structural parameters, control gains and/or weighting constants. A similar problem was also considered by Lust and Schmit (1988). Manning and Schmit (1987) posed the integrated design problem as a composite objective function of total weight and control energy subject to different constraints. The composite objective function is modified to a single objective function, such as control energy (or total weight), with a constraint that the total weight (or control energy) be less than or equal to the specified upper bound, to get rid of the scaling problem.

Rao et al. (1988A, 1988B) investigated the design of actively controlled structures using two multiobjective optimization techniques: goal programming and game theory. The simultaneous structural and control design, with a consideration of the robustness of the controlled structure under uncertainty on parameters, was proposed by Rao et al. (1989). In this paper, the stability and performance robustness indices are introduced. Several multiobjective optimization techniques are used to solve the three-objective problem, the two robustness indices and the total structural weight, to get a more robust controlled structure under certain physical constraints. However, some objective functions may not be unimodal and local optima will be obtained by conventional optimization techniques. Hence revolutionary optimization techniques need to be developed for their solution.

## 1.4 Sensor and Actuator Location Selection Problem

The determination of the number and location of sensors and actuators for distributed flexible structures is an important issue. If the sensors are used not only for the feedback control gain calculation, but also for monitoring the change of system parameters or estimation, the selection of the number and location of the sensor/actuator becomes more critical. Generally speaking, this problem can be solved using optimization techniques or cost decomposition methods. Using optimization methods, the problem can be formulated as:

$$\underset{n,m,\vec{x},\vec{y}}{\text{Min}} f(n,m,\vec{x},\vec{y}) \quad (1.10)$$

subject to the specified constraints, where  $f$  is the cost function which is usually defined as the summation of the output error covariance, input energy and the cost (weight, budget) of sensors and actuators. The variables  $n$  and  $m$  denote the number of sensors and actuators, respectively. The vectors  $\vec{x}$  and  $\vec{y}$  represent the location vectors of sensors and actuators, respectively.

A survey of the field of optimal sensors and/or actuators location for dynamical distributed parameter systems modeled by partial differential equations was presented by Kubrusly and Malebranche (1985). Omatu et al. (1978) solved the optimal sensor location problem for a linear distributed parameter system by using a criterion to minimize the trace of the optimal filtering error covariance function. Schulz and Heimbold (1983) presented an optimization method to determine the locations of sensors, actuators and feedback gains for the control of flexible structures. Heuristic integer programming was used to select the actuator locations in large space structures by Haftka et al. (1985). Salama et al. (1987) cast the location selection problem as a combinatorial optimization problem and solved it by an adaptation of the simulated annealing heuristic algorithm.

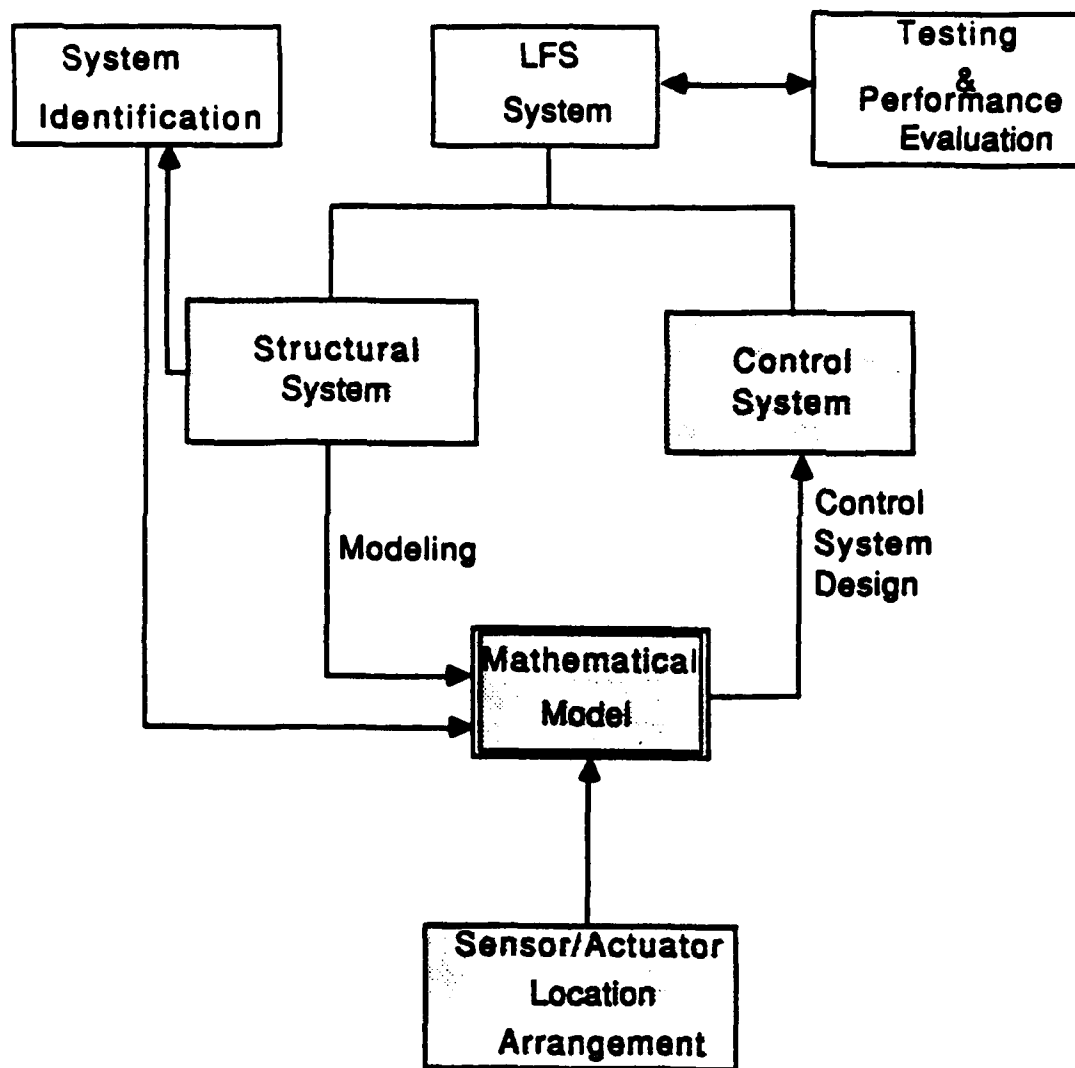
Stieber (1988) investigated the interrelation between the arrangement of sensors and actuators on flexible structures and the hyperstability, which is a system property that guarantees the stability of interconnected systems. Velde and Caignan (1984) considered the number and placement of control system components including possible component failures. It is solved by computing the performance of all admissible combinations of component locations. The placement of sensors in structural control and the associated reliability of the system was analyzed by Baruh and Choe (1988). An optimization technique is used to determine the best locations of the operational sensors. In addition, it is shown that modal filtering by means of spline functions is more desirable for modal coordinate extraction than the other approaches because of stability and robustness considerations. Montgomery and Velde (1985) presented the sensor/actuator location problem with reliability considerations. The optimum placement of sensors and actuators for static deformations and shape control of LSS was investigated by Haftka (1984), and Haftka and Adelman (1987). The actuator/sensor failure detection methods in the control of flexible structures were studied by Baruh (1986),

and Baruh and Choe (1987).

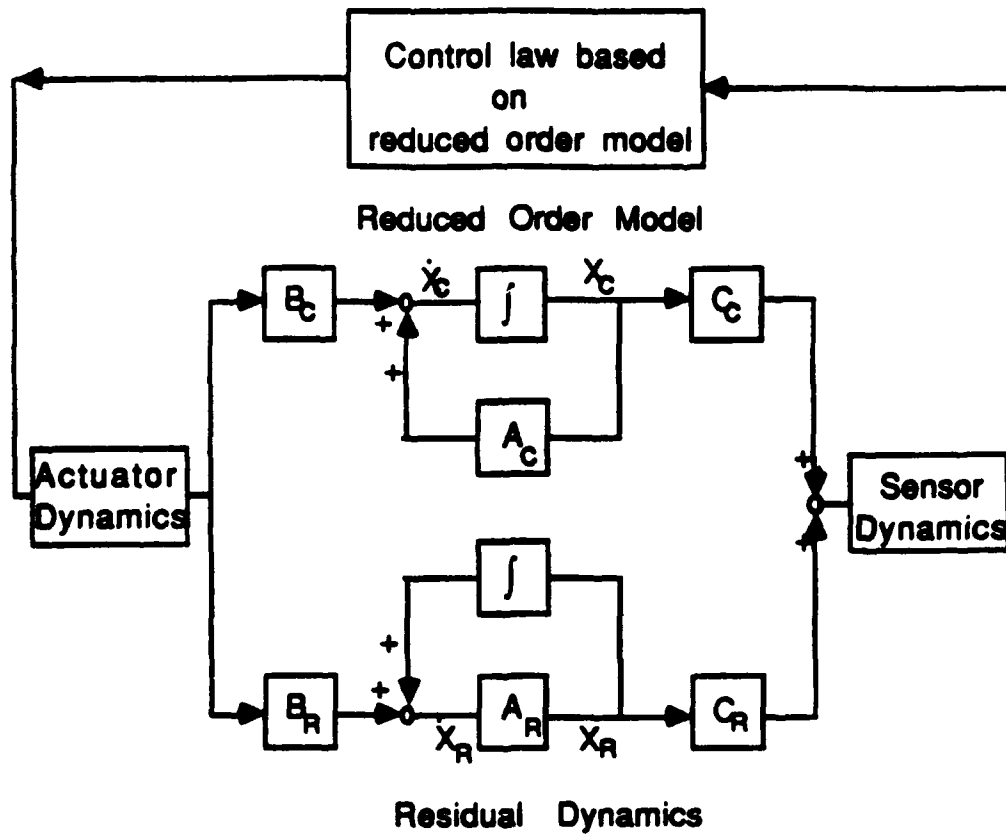
The component cost analysis or input/output cost analysis was developed by Chiu and Skelton (1981) to solve the optimal number and location selection problem in LQG systems. The component cost at each possible location is found and ranked in order. The possible locations having least contribution to the cost function are truncated and the remaining locations are considered to be the best places to locate the sensors/actuators. Additional studies can be found in Chiu and Pan (1983), and Skelton and DeLorenzo (1983). Lindberg and Longman (1984) developed a method to determine the optimal actuator locations for independent modal space control. A projection approach was developed and a sensor location criterion, which minimizes the error caused by the unobservable subspace under nonstationary noise, was derived by Morari and O'Dowd (1980). Other related research was conducted by Jai (1986) and Lafontaine and Stieber (1986). Although many approaches were studied, there is no guarantee to obtain a global optimal solution for the sensor/actuator location selection problem of LFS.

### **1.5 Outline of the Report**

After an overview of the recent research in modeling and control design for flexible structures, the report is organized as follows. Chapter 2 introduces the interacting substructure decentralized control design approach. The idea of the interacting substructure decentralized control originated from the substructure techniques used in structural analysis for large complex structural systems. The interacting force balance plays the key role in the individual control gain design for each substructure. The control gains are made to converge after several iterations of the procedure. This method does not need model reduction and hence the spillover problem is avoided. A passive control design and a dual passive/active control design are introduced in Chapter 3. In Chapter 4, the stability and performance robustness indices are defined, and the integrated structural/control design problem is considered as a multiobjective optimization problem. The optimal actuator/sensor location selection problem is addressed in Chapter 5 and three methods are considered to solve the problem. Finally, Chapter 6 offers some concluding remarks on this research.

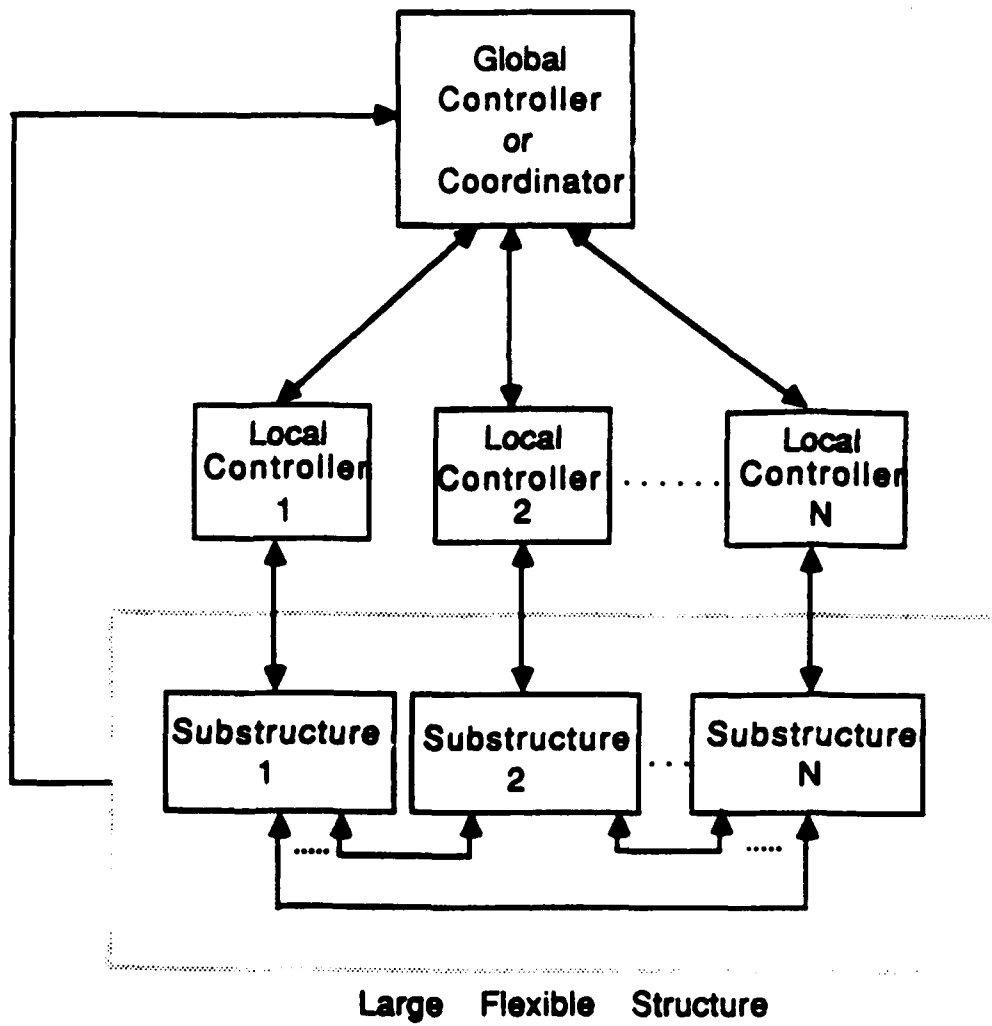


**Figure 1.1 Relationship Between Techniques of Large Flexible Structures**



**Figure 1.2** Model Reduction and Control of Large Flexible Structures





**Figure 1.3** Hierarchical/Decentralized Control of Large Flexible Structures

## CHAPTER 2

### INTERACTING SUBSTRUCTURE DECENTRALIZED CONTROL

#### 2.1 General Formulation of Flexible Structures

For flexible structures, the equations of motion can be expressed as

$$M\ddot{v} + C\dot{v} + Kv = Du \quad (2.1)$$

where  $M, C, K$ , and  $D$  are the mass, damping, stiffness, and the input matrices,  $v$  and  $u$  are the displacement and the input vectors. Letting  $x^T = \{ v^T \dot{v}^T \}$ , Eq.(2.1) can be rewritten as

$$\dot{x} = Ax + Bu \quad (2.2)$$

where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad (2.3)$$

$$B = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix} \quad (2.4)$$

Without loss of generality, the optimal linear quadratic regulator method is applied to design the control gain of the feedback controller for simplicity. Thus, the input vector can be presented as

$$u = -R^{-1}B^TPx \quad (2.5)$$

with  $P$  satisfying the following matrix Riccati equation

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \quad (2.6)$$

where  $Q$  is a positive semidefinite output weighting matrix, and  $R$  is a positive definite input weighting matrix.

#### 2.2 Nodal Condensation for Substructures

### 2.2.1 Direct Condensation

When the number of degrees of freedom in a structure becomes very large, the analysis gets difficult and sometimes becomes impossible. To solve this kind of large structures, one can divide the analytical model into substructures and solve the smaller problems individually. The equation of motion for a specific substructure can be written in terms of the internal and the boundary degrees of freedom as

$$\begin{bmatrix} M_{aa} & M_{ab} \\ M_{ba} & M_{bb} \end{bmatrix} \begin{bmatrix} \ddot{x}_a \\ \ddot{x}_b \end{bmatrix} + \begin{bmatrix} C_{aa} & C_{ab} \\ C_{ba} & C_{bb} \end{bmatrix} \begin{bmatrix} \dot{x}_a \\ \dot{x}_b \end{bmatrix} + \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} = \begin{bmatrix} f_a \\ f_b \end{bmatrix} \quad (2.7)$$

where  $x$  is the displacement vector,  $M$ ,  $C$ , and  $K$  are the mass, damping, and the stiffness matrices, and  $f$  is the force vector. The vectors and the matrices in Eq. (2.7) are partitioned corresponding to the boundary degrees of freedom (subscript  $b$ ) and the internal degrees of freedom (subscript  $a$ ). Let the force term on the right hand side of equation (2.7) be equal to  $\sum_{i=1}^k \bar{f}_i e^{j\omega_i t}$  so that the system response can be expressed as  $\sum_{i=1}^k v_i e^{j\omega_i t}$ . For each force component there is a corresponding response, thus the superposition approach can be applied. The original problem then can be divided into  $k$  equations of motion corresponding to any specified input frequency. Considering, for simplicity, only one force component  $f = \bar{f} e^{j\omega t}$ , Eq.(2.7) can be expressed as

$$\begin{bmatrix} K_{aa} + C_{aa}\omega - M_{aa}\omega^2 & K_{ab} + C_{ab}\omega - M_{ab}\omega^2 \\ K_{ba} + C_{ba}\omega - M_{ba}\omega^2 & K_{bb} + C_{bb}\omega - M_{bb}\omega^2 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} \bar{f}_a \\ \bar{f}_b \end{bmatrix} \quad (2.8)$$

Assume that no force is applied on internal nodes (i.e.,  $\bar{f}_a = 0$ ) so that

$$D_{aa} v_a + D_{ab} v_b = 0 \quad (2.9)$$

where

$$D_{aa} = K_{aa} + C_{aa}\omega - M_{aa}\omega^2 \quad (2.10)$$

$$D_{ab} = K_{ab} + C_{ab}\omega - M_{ab}\omega^2 \quad (2.11)$$

The exact transformation from the degrees of freedom at internal nodes to those of boundary nodes can be obtained by solving Eq.(2.9).

$$v_a = -D_{aa}^{-1} D_{ab} v_b \quad (2.12)$$

Let  $\Phi$  be the fixed interface modal matrix and  $\Omega^2$  be a diagonal matrix consisting of the natural frequencies corresponding to the modal matrix. The following relations between modal data and mass, damping, and stiffness matrices are satisfied.

$$\Phi^T M_{aa} \Phi = \text{Identity Matrix} \quad (2.13)$$

$$\Phi^T K_{aa} \Phi = \Omega^2 \quad (2.14)$$

$$\Phi^T C_{aa} \Phi = 2\Psi \quad (2.15)$$

where  $\Psi$  is damping coefficient matrix. The damping is assumed to be proportional damping so that  $\Psi$  is diagonal. Substituting these relations into Eq.(2.10) yields

$$D_{aa}^{-1} = \Phi(\Omega^2 + 2\Psi\omega - I\omega^2)^{-1} \Phi^T \quad (2.16)$$

In Eq.(2.16), the term in parenthesis is in diagonal form by assuming that all eigenvalues are distinct and hence the inverse of it is the inverse of each diagonal term. For example, the  $i^{\text{th}}$  term can be expressed as

$$\begin{aligned} &(\Omega_i^2 + 2\Psi_i\omega - \omega^2)^{-1} = \\ &\frac{1}{\Omega_i^2} \left\{ 1 - \frac{2\Psi_i}{\Omega_i^2} \omega + \left[ \frac{4\Psi_i^2}{\Omega_i^4} + \frac{1}{\Omega_i^2} \right] \omega^2 \right\} \\ &+ \frac{1}{\Omega_i^2} \left\{ (\Omega_i^2 + 2\Psi_i\omega - I\omega^2)^{-1} \left[ -\frac{8\Psi_i^3}{\Omega_i^6} + \left[ \frac{1}{\Omega_i^4} - \frac{4\Psi_i^2}{\Omega_i^6} \right] \omega \right] \omega^3 \right\} \end{aligned} \quad (2.17)$$

Substituting Eq.(2.17) into Eq.(2.16), the inverse of transfer matrix  $D_{aa}$  can be obtained as follows.

$$D_{aa}^{-1} = K_{aa}^{-1} \left[ 1 - C_{aa} K_{aa}^{-1} \omega + \left[ M_{aa} K_{aa}^{-1} + (C_{aa} K_{aa}^{-1})^2 \right] \omega^2 \right] \\ + \Phi \text{diag} [\text{residual part}] \Phi^T \omega^3 \quad (2.18)$$

where the residual part is contained in the last term of Eq.(2.17). Thus the transformation matrix can be written as

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} -D_{aa}^{-1} D_{ab} \\ I \end{bmatrix} v_b \quad (2.19)$$

This transformation is a function of the frequency of excitation. The frequency content of excitation is usually unknown in advance. If the frequency content of excitation is small compared to the system natural frequencies, the frequency dependent terms in the transfer matrix of Eq.(2.19) will be less important and a simplified transformation can be obtained as follows.

$$D_{aa}^{-1} = K_{aa}^{-1} \quad (2.20)$$

$$D_{ab} = K_{ab} \quad (2.21)$$

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} -K_{aa}^{-1} K_{ab} \\ I \end{bmatrix} v_b \quad (2.22)$$

As a more general case, a serial structure is considered with the equation of motion of each substructure in a tridiagonal form as

$$\begin{bmatrix} M_{B_k B_k} & M_{B_k A_k} & 0 \\ M_{A_k B_k} & M_{A_k A_k} & M_{A_k B_{k+1}} \\ 0 & M_{B_{k+1} A_k} & M_{B_{k+1} B_{k+1}} \end{bmatrix} \begin{bmatrix} \ddot{v}_{B_k} \\ \ddot{v}_{A_k} \\ \ddot{v}_{B_{k+1}} \end{bmatrix} + \begin{bmatrix} C_{B_k B_k} & C_{B_k A_k} & 0 \\ C_{A_k B_k} & C_{A_k A_k} & C_{A_k B_{k+1}} \\ 0 & C_{B_{k+1} A_k} & C_{B_{k+1} B_{k+1}} \end{bmatrix} \begin{bmatrix} \dot{v}_{B_k} \\ \dot{v}_{A_k} \\ \dot{v}_{B_{k+1}} \end{bmatrix}$$

$$+ \begin{bmatrix} K_{B_k B_k} & K_{B_k A_k} & 0 \\ K_{A_k B_k} & K_{A_k A_k} & K_{A_k B_{k+1}} \\ 0 & K_{B_{k+1} A_k} & K_{B_{k+1} B_{k+1}} \end{bmatrix} \begin{bmatrix} v_{B_k} \\ v_{A_k} \\ v_{B_{k+1}} \end{bmatrix} = \begin{bmatrix} F_{B_k} \\ F_{A_k} \\ F_{B_{k+1}} \end{bmatrix} \quad (2.23)$$

in which the subscript  $A_k$  denotes the internal degrees of freedom of substructure  $k$ , and  $B_k$  and  $B_{k+1}$  denote the boundary degrees of freedom between substructure  $k$  and connected substructures  $k-1$  and  $k+1$ , respectively. For a single frequency excitation, Eq.(2.23) can be rewritten as

$$\begin{bmatrix} D_{B_k B_k} & D_{B_k A_k} & 0 \\ D_{A_k B_k} & D_{A_k A_k} & D_{A_k B_{k+1}} \\ 0 & D_{B_{k+1} A_k} & D_{B_{k+1} B_{k+1}} \end{bmatrix} \begin{bmatrix} v_{B_k} \\ v_{A_k} \\ v_{B_{k+1}} \end{bmatrix} = \begin{bmatrix} f_{B_k} \\ f_{A_k} \\ f_{B_{k+1}} \end{bmatrix} \quad (2.24)$$

The internal degrees of freedom can be reduced to boundary degrees of freedom by assuming  $f_{A_k}$  equal to zero:

$$v_{A_k} = -D_{A_k A_k}^{-1} (D_{A_k B_k} v_{B_k} + D_{A_k B_{k+1}} v_{B_{k+1}}) \quad (2.25)$$

Thus the transformation can be obtained as

$$\begin{bmatrix} v_{B_k} \\ v_{A_k} \\ v_{B_{k+1}} \end{bmatrix} = T_k \begin{bmatrix} v_{B_k} \\ v_{B_{k+1}} \end{bmatrix} = \begin{bmatrix} I_{B_k} & 0 \\ -D_{A_k A_k}^{-1} D_{A_k B_k} & -D_{A_k A_k}^{-1} D_{A_k B_{k+1}} \\ 0 & I_{B_{k+1}} \end{bmatrix} \begin{bmatrix} v_{B_k} \\ v_{B_{k+1}} \end{bmatrix} \quad (2.26)$$

where

$$\begin{aligned} D_{A_k A_k} &= K_{A_k A_k} + C_{A_k A_k} \omega - M_{A_k A_k} \omega^2 \\ D_{A_k B_k} &= K_{A_k B_k} + C_{A_k B_k} \omega - M_{A_k B_k} \omega^2 \\ D_{A_k B_{k+1}} &= K_{A_k B_{k+1}} + C_{A_k B_{k+1}} \omega - M_{A_k B_{k+1}} \omega^2 \end{aligned} \quad (2.27)$$

The transformation matrix can be reduced to a simple form if the frequency content of

excitation is small compared to the system natural frequencies. Thus

$$T_k = \begin{bmatrix} I_{B_k} & 0 \\ -K^{-1}_{A_k A_k} K_{A_k B_k} & -K^{-1}_{A_k A_k} K_{A_k B_{k+1}} \\ 0 & I_{B_{k+1}} \end{bmatrix} \quad (2.28)$$

### 2.2.2 Indirect Condensation

An alternative condensation approach, using modal data of each substructure, is developed in this section. The full order modal matrix ( $\Phi$ ) need to be found in this approach and the modal coordinates are denoted by the vector  $y$ . The displacement  $v$  can then be represented as

$$v = \Phi y \quad (2.29)$$

and the inverse of  $\Phi$  exists, so that

$$y = \Phi^{-1} v \quad (2.30)$$

Suppose that the modal coordinate vector  $y$  is arranged in two parts which are truncated modes ( $y_t$ ) and retaining modes ( $y_r$ ). The physical coordinates ( $v$ ) are also arranged in two parts which are internal degrees of freedom ( $v_a$ ) and boundary degrees of freedom ( $v_b$ ). Then Eq.(2.30) can be rewritten in the following form

$$\begin{Bmatrix} y_t \\ y_r \end{Bmatrix} = \begin{bmatrix} \bar{\Phi}_{ta} & \bar{\Phi}_{tb} \\ \bar{\Phi}_{ra} & \bar{\Phi}_{rb} \end{bmatrix} \begin{Bmatrix} v_a \\ v_b \end{Bmatrix} \quad (2.31)$$

where

$$\Phi^{-1} = \begin{bmatrix} \bar{\Phi}_{ta} & \bar{\Phi}_{tb} \\ \bar{\Phi}_{ra} & \bar{\Phi}_{rb} \end{bmatrix} \quad (2.32)$$

In Eq.(2.31), the number of truncated modes will be chosen same as the number of internal degrees of freedom. To eliminate the truncated modes, vector  $y_t$  is set to be zero and hence

$$\bar{\Phi}_{ta} v_a + \bar{\Phi}_{tb} v_b = 0 \quad (2.33)$$

The relation between  $v_a$  and  $v_b$  then can be obtained as

$$v_a = -\bar{\Phi}_{ta}^{-1} \bar{\Phi}_{tb} v_b \quad (2.34)$$

and

$$\begin{Bmatrix} v_a \\ v_b \end{Bmatrix} = \begin{bmatrix} T_{ab} \\ I \end{bmatrix} v_b \quad (2.35)$$

where

$$T_{ab} = -\bar{\Phi}_{ta}^{-1} \bar{\Phi}_{tb} \quad (2.36)$$

### 2.3 Interacting Substructure Decentralized Control for Serial LFS

For a large-scale interconnected system composed of a number of subsystems or for a system distributed widely in space, the scheme of decentralized control has advantages in computation and implementation of control laws (Siljak, 1976, Sandell, Jr. et al., 1978, and Singh, 1981). When decentralized optimal control is used, the resulting closed-loop system is guaranteed to have robust stability properties against variations in the open-loop dynamics (Ikeda et al., 1983). Hence the stabilization of large-scale systems by means of local state feedback has been applied extensively.

In this section, a new decentralized control approach for serial LFS is developed by using interacting substructure technique. The optimal linear quadratic regulator method is used for the controller design. Let the complete flexible structure be divided into  $n_s$  substructures as indicated in Fig. 2.1 and the equations of motion of the  $k^{\text{th}}$  substructure be given by Eq.(2.7) which can be rewritten in state form as in Eq.(2.2). The controller design starts from the first substructure which contains nodes belonging to the global boundary as well as the local boundary between substructures. Without loss of generality, optimal LQR method is used in control design for simplicity. In the first iteration, the controller is designed by assuming the local boundary to be free. The state equation and the control law can be written as follows:

$$\dot{x}_1 = A_1 x_1 + B_1 u_1 \quad (2.37)$$

The optimal state feedback gain is given by

$$u_1 = -G_1 x_1 = -R_1^{-1} B_1^T P_1 x_1 \quad (2.38)$$

where  $P_1$  satisfies the matrix Riccati equation

$$A_1^T P_1 + P_1 A_1 - P_1 B_1 R_1^{-1} B_1^T P_1 + Q_1 = 0 \quad (2.39)$$

The state vector  $x_1$  can be divided into interior ( $x_{1A}$ ) and boundary ( $x_{1B}$ ) states so that,

$$\dot{x}_1 = (\dot{x}_{A_1}^T \dot{x}_{B_1}^T \ddot{x}_{A_1}^T \ddot{x}_{B_1}^T)^T = A_1 x_1 + B_1 u_1$$



$$= A_1 x_1 + \begin{bmatrix} 0 \\ -\bar{B}_1 G_1 \end{bmatrix} x_1 \quad (2.40)$$

where  $\bar{B}_1$  is lower half part of matrix  $B_1$ . The control force can be obtained by multiplying  $-\bar{B}_1 G_1$  by  $M_1$  and expressing in the following form:

$$\begin{bmatrix} f_{A_1} \\ f_{B_1} \end{bmatrix} = -M_1 \bar{B}_1 G_1 x_1 \quad (2.41)$$

where  $f_{A_1}$  and  $f_{B_1}$  are equivalent forces applied on internal and boundary degrees of freedom. Applying the transformation in Eq.(2.22) gives

$$\begin{bmatrix} f_{A_1} \\ f_{B_1} \end{bmatrix} = -M_1 \bar{B}_1 G_1 \begin{bmatrix} T_{A_1 B_1} & 0 \\ I & 0 \\ 0 & T_{A_1 B_1} \\ 0 & I \end{bmatrix} \begin{bmatrix} x_{B_1} \\ \dot{x}_{B_1} \end{bmatrix} \quad (2.42)$$

where

$$T_{A_1 B_1} = -K^{-1} A_1 A_1 K_{A_1 B_1} \quad (2.43)$$

The equivalent forces applied on boundary degrees of freedom can be expressed as  $F_B$

$$F_B = \begin{bmatrix} 0 & I_{B_1} \end{bmatrix} \begin{bmatrix} f_{A_1} \\ f_{B_1} \end{bmatrix} = f_{1B_1} x_{B_1} + f_{2B_1} \dot{x}_{B_1} \quad (2.44)$$

where  $I_{B_1}$  is the identity matrix with the same dimension of  $x_{B_1}$ .

The intermediate substructure  $k$  ( $k \neq 1$  and  $n_s$ ) will have boundaries connected to the  $(k-1)^{th}$  and the  $(k+1)^{th}$  substructures. Let the corresponding boundary states be represented as  $x_{B_k}$  and  $x_{B_{k+1}}$  and the boundary force contributions be denoted as  $F_{B_k}$  and  $F_{B_{k+1}}$ , respectively. The equations of motion of the  $k^{th}$  substructure are given by

$$\ddot{x}_k = \begin{bmatrix} \ddot{x}_{B_k}^T & \ddot{x}_{A_k}^T & \ddot{x}_{B_{k+1}}^T & \ddot{x}_{B_k}^T & \ddot{x}_{A_k}^T & \ddot{x}_{B_{k+1}}^T \end{bmatrix}^T$$

$$= A_k x_k + B_k u_k + \begin{bmatrix} 0 \\ M_k^{-1} F_{B_k} \end{bmatrix} + \begin{bmatrix} 0 \\ M_k^{-1} F_{B_{k+1}} \end{bmatrix} \quad (2.45)$$

In the first iteration,  $F_{B_{k+1}}$  ( $k=1$  to  $n_s-1$ ) is assumed to be a zero matrix so that the boundary forces can be explicitly expressed as

$$F_{B_k} = \begin{bmatrix} f_{1B_k} x_{B_k} + f_{2B_k} \dot{x}_{B_k} \\ 0 \\ 0 \end{bmatrix} \quad (2.46)$$

and

$$F_{B_{k+1}} = \begin{bmatrix} 0 \\ 0 \\ f_{1B_{k+1}} x_{B_{k+1}} + f_{2B_{k+1}} \dot{x}_{B_{k+1}} \end{bmatrix} \quad (2.47)$$

Substituting Eqs.(2.46) and (2.47) into Eq.(2.45) gives

$$\begin{aligned} \dot{x}_k &= (A_k + \bar{F}_{B_k} + \bar{F}_{B_{k+1}}) x_k + B_k u_k \\ &= \bar{A}_k x_k + B_k u_k \end{aligned} \quad (2.48)$$

where

$$\bar{F}_{B_k} x_k = \begin{bmatrix} 0 \\ M_k^{-1} F_{B_k} \end{bmatrix} \quad (2.49)$$

$$\bar{F}_{B_{k+1}} x_k = \begin{bmatrix} 0 \\ M_k^{-1} F_{B_{k+1}} \end{bmatrix} \quad (2.50)$$

The optimal linear quadratic regulator method is applied to solve the constant optimal feedback

gain for substructure k:

$$u_k = -G_k x_k = -R_k^{-1} B_k^T P_k x_k \quad (2.51)$$

where  $P_k$  satisfies the Riccati equation

$$\bar{A}_k^T P_k + P_k \bar{A}_k - P_k B_k R_k^{-1} B_k^T P_k + Q_k = 0 \quad (2.52)$$

Similarly, the forces applied on the boundary which are contributed by the actuators involved in the substructure k can be formed as

$$B_k u_k = \begin{bmatrix} 0 \\ -\bar{B}_k G_k \end{bmatrix} x_k \quad (2.53)$$

where  $\bar{B}_k$  is the lower half part of matrix  $B_k$ . Multiplying  $-\bar{B}_k G_k$  by the mass matrix gives the control force vector:

$$\begin{bmatrix} F_{B_k} \\ F_{A_k} \\ F_{B_{k+1}} \end{bmatrix} = -M_k \bar{B}_k G_k x_k = -M_k \bar{B}_k G_k \bar{T}_k \begin{bmatrix} x_{B_k} \\ x_{B_{k+1}} \\ \dot{x}_{B_k} \\ \dot{x}_{B_{k+1}} \end{bmatrix} \quad (2.54)$$

where  $\bar{T}_k$  can be obtained from Eq.(2.28) for direct condensation or Eqs.(2.35) and (2.36) for indirect condensation as:

$$\bar{T}_k = \begin{bmatrix} T_k & 0 \\ 0 & T_k \end{bmatrix} \quad (2.55)$$

The forces acting on the boundary degrees of freedom are given by  $F_{B_k}$  and  $F_{B_{k+1}}$  in Eq.(2.54). In order to include boundary forces into the equations of motion of connected substructures, the equivalent boundary forces should be transformed to be functions which depend on its own boundary degrees of freedom. From direct condensation Eq.(2.24), the first and the third equations give

$$D_{B_k B_k} v_{B_k} + D_{B_k A_k} v_{A_k} = F_{B_k} \quad (2.56)$$

$$D_{B_{k+1} A_k} v_{A_k} + D_{B_{k+1} B_{k+1}} v_{B_{k+1}} = F_{B_{k+1}} \quad (2.57)$$

Substituting  $v_{A_k}$  from Eq.(2.25) gives

$$(D_{B_k B_k} + D_{B_k A_k} T_{A_k B_k}) v_{B_k} + D_{B_k A_k} T_{A_k B_{k+1}} v_{B_{k+1}} = F_{B_k} \quad (2.58)$$

$$D_{B_{k+1} A_k} T_{A_k B_k} v_{B_k} + (D_{B_{k+1} B_{k+1}} + D_{B_{k+1} A_k} T_{A_k B_{k+1}}) v_{B_{k+1}} = F_{B_{k+1}} \quad (2.59)$$

where

$$T_{A_k B_k} = -D^{-1}_{A_k A_k} D_{A_k B_k} \quad (2.60)$$

Assuming  $F_{B_k}$  or  $F_{B_{k+1}}$  to be zero gives the relationship between the displacement vectors  $v_{B_k}$  and  $v_{B_{k+1}}$ :

$$v_{B_k} = -(D_{B_k B_k} + D_{B_k A_k} T_{A_k B_k})^{-1} D_{B_k A_k} T_{A_k B_{k+1}} v_{B_{k+1}} = T_{B_k B_{k+1}} v_{B_{k+1}} \quad (2.61)$$

$$v_{B_{k+1}} = -(D_{B_{k+1} B_{k+1}} + D_{B_{k+1} A_k} T_{A_k B_{k+1}})^{-1} D_{B_{k+1} A_k} T_{A_k B_k} v_{B_k} = T_{B_{k+1} B_k} v_{B_k} \quad (2.62)$$

The transfer matrices in Eqs.(2.61) and (2.62) can be reduced to the corresponding stiffness matrices if excitation contains only low (compared to the system natural frequency) frequency components (i.e.,  $D$  is substituted by  $K$  in Eqs.(2.61) and (2.62)). Thus the equivalent boundary forces can be rewritten as

$$F_{B_k} = - \begin{bmatrix} I_{B_k} & 0 & 0 \end{bmatrix} M_k \bar{B}_k G_k T_k \bar{T}_{k+1,k} \begin{bmatrix} x_{B_k} \\ \dot{x}_{B_k} \end{bmatrix} \quad (2.63)$$

$$F_{B_{k+1}} = - \begin{bmatrix} 0 & 0 & I_{B_{k+1}} \end{bmatrix} M_k \bar{B}_k G_k T_k \bar{T}_{k,k+1} \begin{bmatrix} x_{B_{k+1}} \\ \dot{x}_{B_{k+1}} \end{bmatrix} \quad (2.64)$$

where

$$\bar{T}_{k+1,k} = \begin{bmatrix} I_{B_k} & 0 \\ T_{B_{k+1}B_k} & 0 \\ 0 & I_{B_k} \\ 0 & T_{B_{k+1}B_k} \end{bmatrix} \quad (2.65)$$

and

$$\bar{T}_{k,k+1} = \begin{bmatrix} T_{B_kB_{k+1}} & 0 \\ I_{B_{k+1}} & 0 \\ 0 & T_{B_kB_{k+1}} \\ 0 & I_{B_{k+1}} \end{bmatrix} \quad (2.66)$$

Equations (2.63) and (2.64) give two equivalent forces which are contributed by the actuators of substructure  $k$  and act on the boundary nodes of substructures  $(k-1)$  and  $(k+1)$ , respectively. The transformation matrix for indirect condensation approach is much simpler. For the interacting force on boundary  $B_{k+1}$ , only the degrees of freedom belonging to  $B_{k+1}$  are considered as boundary d.o.f.'s. Hence, the transformation matrix in Eqs.(2.35) and (2.36) are valid after proper assignment of interior and boundary degrees of freedom. The controller of each substructure can be designed one after the other sequentially. In the first iteration, the acting force contributed by the connected substructures is on one side only (except the first substructure). In the consecutive iterations, all substructures, except the first and the last, are subject to interacting forces on both sides. The control gain of each substructure converges iteratively. Proper summation then can be used to obtain the global control gain corresponding to the complete system state  $x_s$  which is the union of  $x_1, x_2, \dots, x_n$ .

## 2.4 ISDC for Substructures Containing No Actuators

The basic idea of ISDC discussed in the last section is the interacting force balancing. If there is any substructure containing no actuator, the interacting force to the connected substructures is assumed to be zero which can not balance with those forces from connected substructures. Hence the procedure described in the last section is not valid any more. To generalize the ISDC procedure, interface stiffness and inertia loading, used in the dynamic analysis of large complex structures, is used to connect the substructures containing no actuators. For example, two connected substructures 1 and 2 have the same boundary (i.e.,  $v_{B_1} = v_{B_2}$ ) and the equation of motion (EOM) of substructure 2 is influenced by substructure 1.

Thus, the modified EOM of substructure 2 can be written as:

$$\begin{bmatrix} M_{A_2A_2} & M_{A_2B_2} \\ M_{B_2A_2} & M_{B_2B_2} + \bar{M}_{B_1} \end{bmatrix} \begin{bmatrix} \ddot{v}_{A_2} \\ \ddot{v}_{B_2} \end{bmatrix} + \begin{bmatrix} C_{A_2A_2} & C_{A_2B_2} \\ C_{B_2A_2} & C_{B_2B_2} + \bar{C}_{B_1} \end{bmatrix} \begin{bmatrix} \dot{v}_{A_2} \\ \dot{v}_{B_2} \end{bmatrix} + \begin{bmatrix} K_{A_2A_2} & K_{A_2B_2} \\ K_{B_2A_2} & K_{B_2B_2} + \bar{K}_{B_1} \end{bmatrix} \begin{bmatrix} v_{A_2} \\ v_{B_2} \end{bmatrix} = \begin{bmatrix} f_{A_2} \\ f_{B_2} \end{bmatrix} \quad (2.67)$$

where

$$\bar{M}_{B_1} = T_{12}^T M_1 T_{12} \quad (2.68)$$

$$\bar{C}_{B_1} = T_{12}^T C_1 T_{12} \quad (2.69)$$

$$\bar{K}_{B_1} = T_{12}^T K_1 T_{12} \quad (2.70)$$

and the transformation matrix  $T_{12}$  can be obtained from Eq.(2.22) as follows:

$$T_{12} = \begin{bmatrix} -(K_{A_1A_1})^{-1} K_{A_1B_1} \\ I \end{bmatrix} \quad (2.71)$$

where  $M_1$ ,  $C_1$ , and  $K_1$  are the mass, damping and the stiffness matrices of substructure 1.

For a serial structure, a more general form can be derived for substructure  $k$ :

$$\begin{bmatrix} M_{B_kB_k} + \bar{M}_{B_k}^{k-1} & M_{B_kA_k} & 0 \\ M_{A_kB_k} & M_{A_kA_k} & M_{A_kB_{k+1}} \\ 0 & M_{B_{k+1}A_k} & M_{B_{k+1}B_{k+1}} + \bar{M}_{B_{k+1}}^{k+1} \end{bmatrix} \begin{bmatrix} \ddot{v}_{B_k} \\ \ddot{v}_{A_k} \\ \ddot{v}_{B_{k+1}} \end{bmatrix}$$

$$\begin{aligned}
& + \begin{bmatrix} C_{B_k B_k} + \bar{C}^{k-1}_{B_k} & C_{B_k A_k} & 0 \\ C_{A_k B_k} & C_{A_k A_k} & C_{A_k B_{k+1}} \\ 0 & C_{B_{k+1} A_k} & C_{B_{k+1} B_{k+1}} + \bar{C}^{k+1}_{B_{k+1}} \end{bmatrix} \begin{bmatrix} \dot{v}_{B_k} \\ \dot{v}_{A_k} \\ \dot{v}_{B_{k+1}} \end{bmatrix} \\
& + \begin{bmatrix} K_{B_k B_k} + \bar{K}^{k-1}_{B_k} & K_{B_k A_k} & 0 \\ K_{A_k B_k} & K_{A_k A_k} & K_{A_k B_{k+1}} \\ 0 & K_{B_{k+1} A_k} & K_{B_{k+1} B_{k+1}} + \bar{K}^{k+1}_{B_{k+1}} \end{bmatrix} \begin{bmatrix} v_{B_k} \\ v_{A_k} \\ v_{B_{k+1}} \end{bmatrix} = \begin{bmatrix} F_{B_k} \\ F_{A_k} \\ F_{B_{k+1}} \end{bmatrix} \quad (2.72)
\end{aligned}$$

where

$$\bar{M}^{k-1}_{B_k} = T_{k-1,k}^T M_{k-1} T_{k-1,k} \quad (2.73)$$

$$\bar{M}^{k+1}_{B_{k+1}} = T_{k+1,k+1}^T M_{k+1} T_{k+1,k+1} \quad (2.74)$$

$$\bar{C}^{k-1}_{B_k} = T_{k-1,k}^T C_{k-1} T_{k-1,k} \quad (2.75)$$

$$\bar{C}^{k+1}_{B_{k+1}} = T_{k+1,k+1}^T C_{k+1} T_{k+1,k+1} \quad (2.76)$$

$$\bar{K}^{k-1}_{B_k} = T_{k-1,k}^T K_{k-1} T_{k-1,k} \quad (2.77)$$

$$\bar{K}^{k+1}_{B_{k+1}} = T_{k+1,k+1}^T K_{k+1} T_{k+1,k+1} \quad (2.78)$$

Transformation matrices  $T_{k-1,k}$  and  $T_{k+1,k+1}$  can be derived from Eqs.(2.28), (2.61), and (2.62) and expressed as

$$T_{k-1,k} = \begin{bmatrix} I & & \\ & -(K_{A_{k-1}A_{k-1}})^{-1} K_{A_{k-1}B_k} & \\ (K_{B_{k-1}B_{k-1}} - K_{B_{k-1}A_{k-1}} K^{-1}_{A_{k-1}A_{k-1}} K_{A_{k-1}B_{k-1}})^{-1} K_{B_{k-1}A_{k-1}} K^{-1}_{A_{k-1}A_{k-1}} K_{A_{k-1}B_k} & & \\ & \ddots & \end{bmatrix} \quad (2.79)$$

$$T_{k+1,k+1} = \begin{bmatrix} (K_{B_{k+2}B_{k+2}} - K_{B_{k+2}A_{k+1}} K^{-1}_{A_{k+1}A_{k+1}} K_{A_{k+1}B_{k+2}})^{-1} K_{B_{k+2}A_{k+1}} K^{-1}_{A_{k+1}A_{k+1}} K_{A_{k+1}B_{k+1}} & & \\ & -K^{-1}_{A_{k+1}A_{k+1}} K_{A_{k+1}B_{k+1}} & \\ & & I \\ & \ddots & \end{bmatrix} \quad (2.80)$$

Thus the substructures which contain no actuators can be condensed to the connected substructures using the equivalent stiffness and inertial loading. The interacting force used in ISDC approach can then be transmitted through the substructure containing no actuator to the other substructures. For example, let the  $k^{\text{th}}$  substructure be the only substructure containing no actuator. Then the modified ISDC procedure can be summarized as follows.

1. Use the ISDC procedure of section 2.3.
2. Modify the equation of motion of substructure (k-1) by adding the equivalent dynamic loading, which includes the interface stiffness, inertial loading and interacting force from substructure (k+1), of substructure k and interacting force from substructure (k-2).
3. Design the controller using the modified EOM of substructure (k-1).
4. Similarly, modify the equation of motion of substructure (k+1) by adding the equivalent dynamic loading of substructure k and interacting force from substructure (k+2).
5. Use the ISDC procedure of section 2.3.

The simple flowchart for the general ISDC procedure is presented in Figure 2.2.

## 2.5 ISDC for General Structures

For more general structures, any substructure might connect with more than two substructures. The direct condensation process will lead to a difficulty in formulation of the transformation matrix. The indirect condensation process can express the transformation matrix as in sec. (2.2.2) in which interior degrees of freedom include the original interior d.o.f.'s and all other boundary d.o.f.'s which are not needed in the final expression. For example, a substructure k, which connects m substructures, contains the original interior degrees of



freedom ( $v_a$ ) and the boundary degrees of freedom ( $v_{b_i}$ ) which connect substructures  $k$  and  $i$  ( $i=k1, k2, k3, \dots, km$ ). If the transformation matrix between  $v_{b_{k1}}$  and all other d.o.f.'s is going to be found then the interior d.o.f.'s in Eq.(2.35) consist of  $v_a$  and  $v_{b_j}$  ( $j=k2, k3, \dots, km$ ), and the boundary d.o.f.'s consist of  $v_{k1}$ . Similarly, the transformation matrix between each set of boundary d.o.f.'s and the other d.o.f.'s can be found. Hence, the indirect condensation process makes the transformation very simple and can be used in any general structure.

## 2.6 Examples

Two examples, involving a two-bay truss and a six-bay truss, are considered to illustrate the procedure of interacting substructure decentralized control outlined in this chapter. For the two-bay truss (Fig. 2.3), two substructures are used and four sensors and actuators are colocated at degrees of freedom 1, 2, 5, and 6. The weighting matrices  $Q$  and  $R$  are selected as  $10^5 \cdot I$  and  $I$ , respectively, where  $I$  is the identity matrix. Following the procedure described in sec. 2.3, the closed-loop eigenvalues obtained are shown in Fig.2.4 which are compared with those of the controlled structure designed using the complete system model. Most of the closed-loop eigenvalues are close for these two systems. Furthermore, the first three modes for the system with ISDC design have better damping property than the system with LQR controller. The simulation results for the dynamic responses of degrees of freedom 1, 2, 5, and 6 under a unit impulse at degrees of freedom 1 and 2 is shown in Figs.2.5 to 2.8. These results indicate that the interacting substructure decentralized control algorithm leads to a better dynamic response. It doesn't mean that the ISDC approach is better than the LQR design because the performance index might not be better for the system with the ISDC design.

For the six-bay truss (Fig.2.9), three substructures are used and six sensors and actuators are colocated at degrees of freedom 1, 2, 5, 6, 17, and 18. The weighting matrices  $Q$  and  $R$  are chosen as  $10^5 \cdot I$  and  $I$ , respectively. The global controller is designed using Eqs.(2.37)-(2.66). The closed-loop eigenvalues are plotted in Fig.2.10 and compared with those of the LQR controller designed using the complete system model. The ISDC design gives a more conservative result than the LQR controller with complete system model for lower frequency modes. Figures 2.11 to 2.16 show the simulation results of degrees of freedom 1, 2, 5, 6, 17, and 18 under a unit impulse at degrees of freedom 1 and 2 (horizontal and vertical directions at tip of truss). A better dynamic response is shown for the controller designed by the ISDC approach. The input forces of the six actuators are illustrated in Figures 2.17 to 2.22. The maximum amplitudes of the first four actuators are almost the same, while those of the fifth and the sixth actuators of the ISDC controller are about 15% to 25% higher than those of LQR controller designed with complete system model.

The performance index,  $\int_0^t (x^T Q x + u^T R u) dt$ , is presented in Fig.2.23 for both the systems. The performance index of the system with ISDC controller is higher than that of the system

with LQR controller by 17.7% for the case of impulse input. The second case of simulation involves applying the initial displacements to the various degrees of freedom, which are obtained by applying a static force at node 1 in vertical direction such that the displacement at node 1 is unity. The dynamic responses of degrees of freedom 1,2,5,6,17, and 18 are shown in Figures 2.24 to 2.29. The dynamic response of the system with ISDC controller is a little better than that of the system with LQR controller. Figures 2.30 to 2.35 show the input time history of both systems. They show that the ISDC approach induces smaller input forces for the first three actuators but higher input forces for the last three actuators. In Fig. 2.36, the performance index of the system with ISDC controller is about 20.1% higher than that of the system with LQR controller.

For the six-bay truss, the actuators are rearranged at d.o.f.'s 1,2,5,6,21, and 22 and three substructures are arranged as shown in Fig. 2.9. Here the second substructure contains no actuator, and the design procedure of sec. 2.4 is then used to solve the control design problem. The resulting closed-loop eigenvalues are compared with those of the optimal design in Figure 2.37.

The ISDC procedures with direct and indirect condensation approaches are compared in Figure 2.38 for the six-bay truss described in Fig. 2.9. The closed-loop system eigenvalues obtained using the indirect condensation approach are closer to those of the optimal design than those using the direct condensation approach. This shows that the indirect condensation approach is better than the direct condensation approach.

## 2.7 Concluding Remarks

The interacting substructure decentralized control design procedure has been developed for large flexible structural control. This method is based on the physical coordinates instead of modal coordinates. Thus, the natural frequency and modal matrix information is not required and no model data error is introduced into the system. By using the concept of interacting substructures, the size of the mathematical model can be reduced without using model reduction techniques and hence the "spillover problem" can be avoided. During the nodal condensation process, two approaches, direct and indirect condensation processes, are considered. Although no modal data is involved in the direct condensation approach, it can only be applied to serial structures. While the indirect condensation involves substructure modal data, it can be applied to any general structure. As can be expected, the design of the ISDC using indirect condensation approach will be closer to the optimal design by increasing the number of modes used in finding the transformation matrix, which can be done by increasing the number of boundary degrees of freedom when the substructures are defined. Generally speaking, the computational efficiency of the ISDC approach will be more significant when it is applied to very large flexible structures.

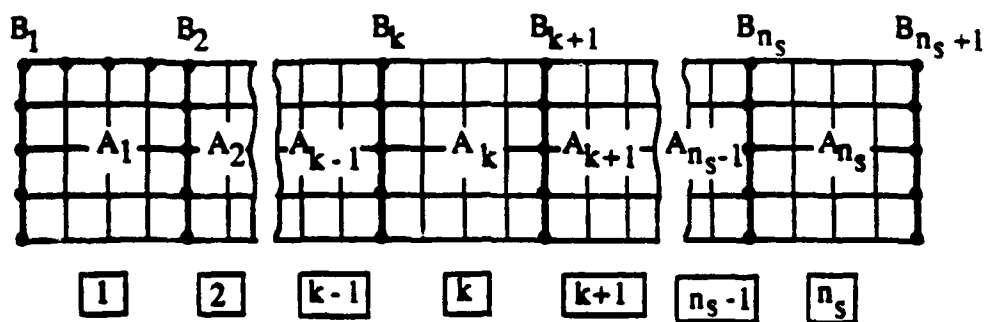


Figure 2.1 Substructures in Series

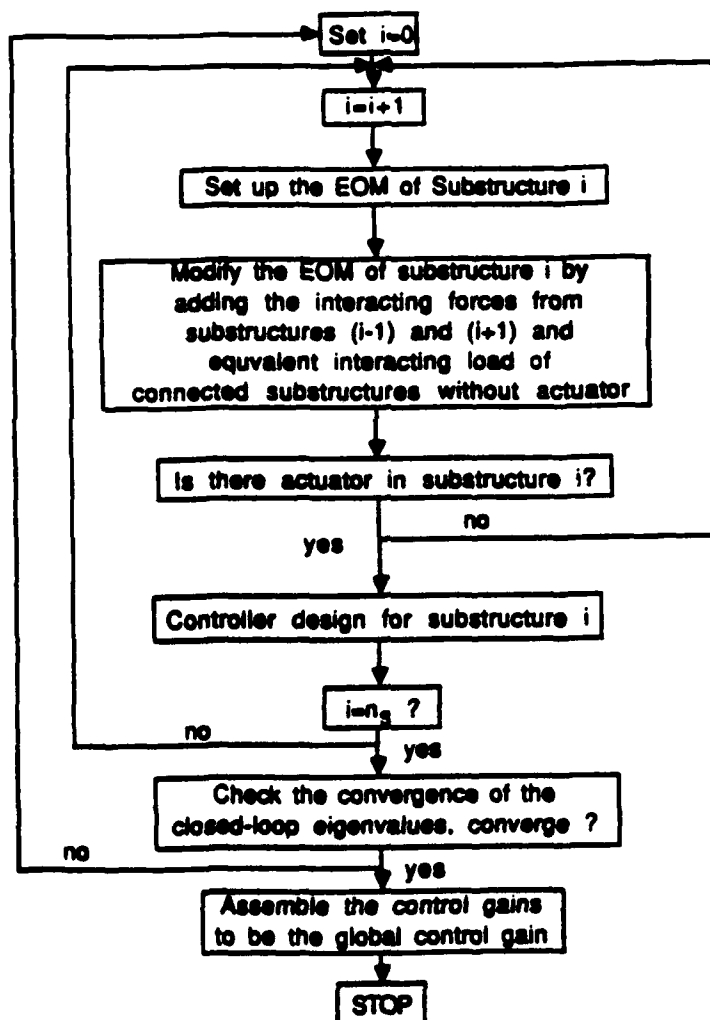
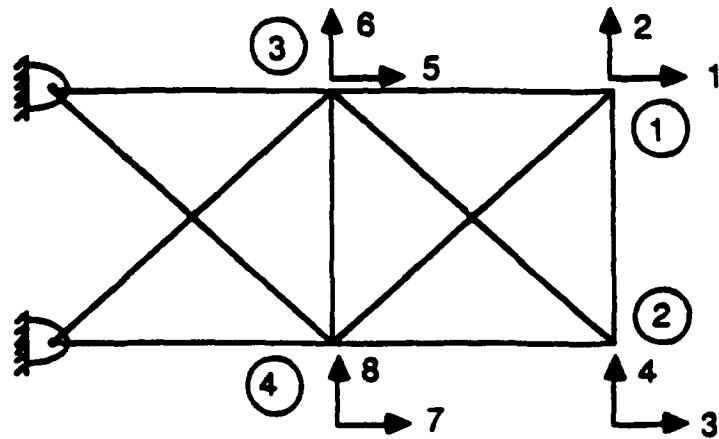
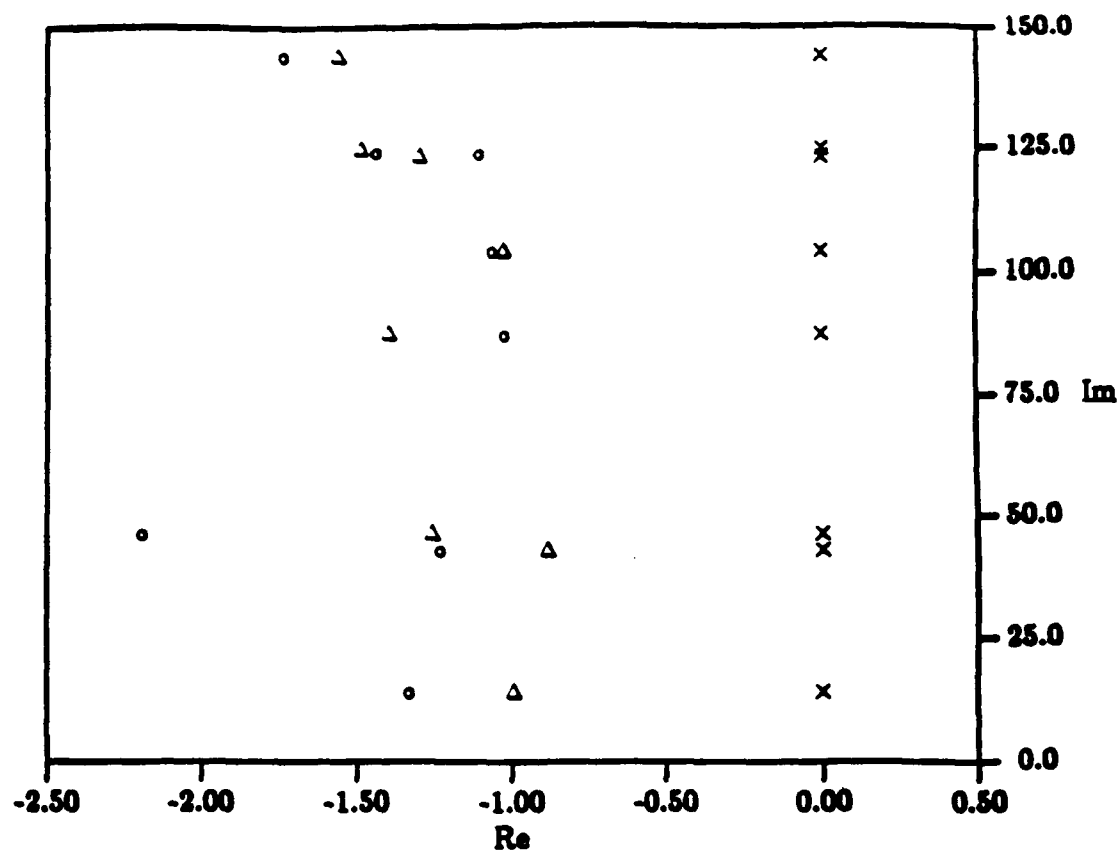


Figure 2.2 General ISDC Procedure for Serial Large Flexible Structures



Note: Numbers represent degree of freedom  
 Numbers with circle represent the node numbers

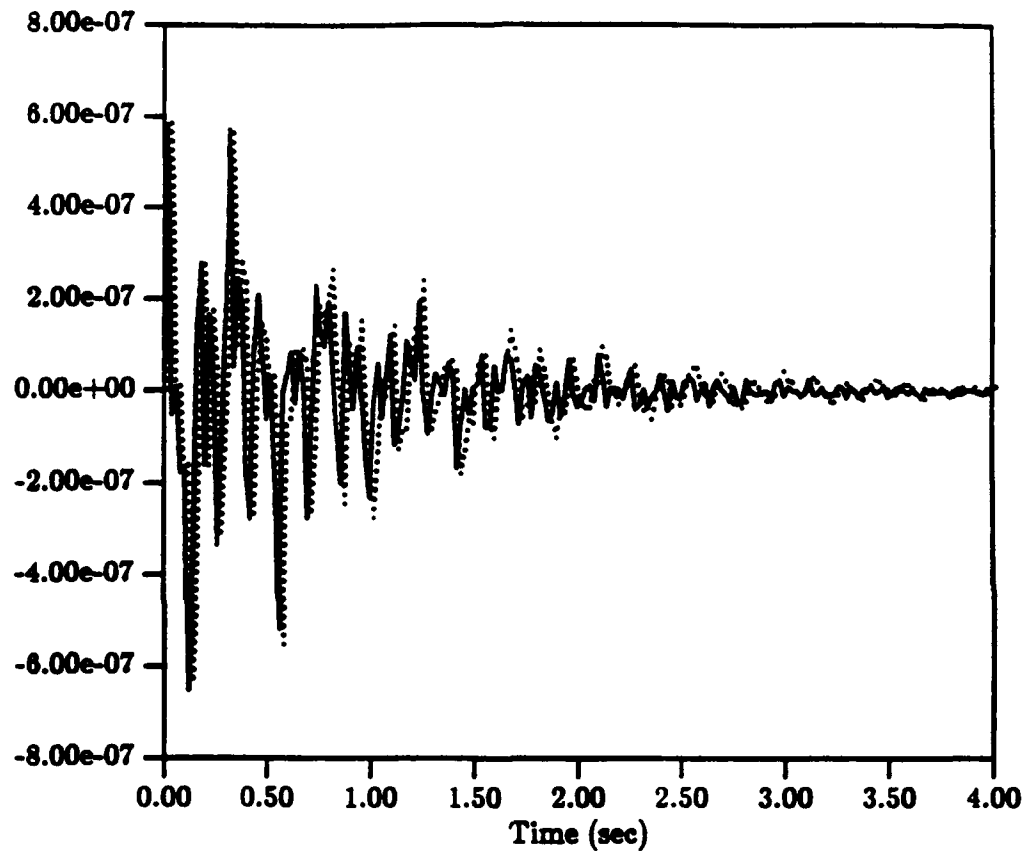
Figure 2.3 Two-Bay Truss



**Legend :**

- ×** : Eigenvalues of open-loop structure
- Δ** : Eigenvalues of closed-loop system with controller designed with complete model
- : Eigenvalues of closed-loop system with controller designed with substructure model

**Figure 2.4 Eigenvalues of Two-Bay Truss**



—— : Controller designed with substructure model  
..... : Controller designed with complete model

Figure 2.5 Displacement of 2-bay Truss at d.o.f. 1 (impulse at tip)

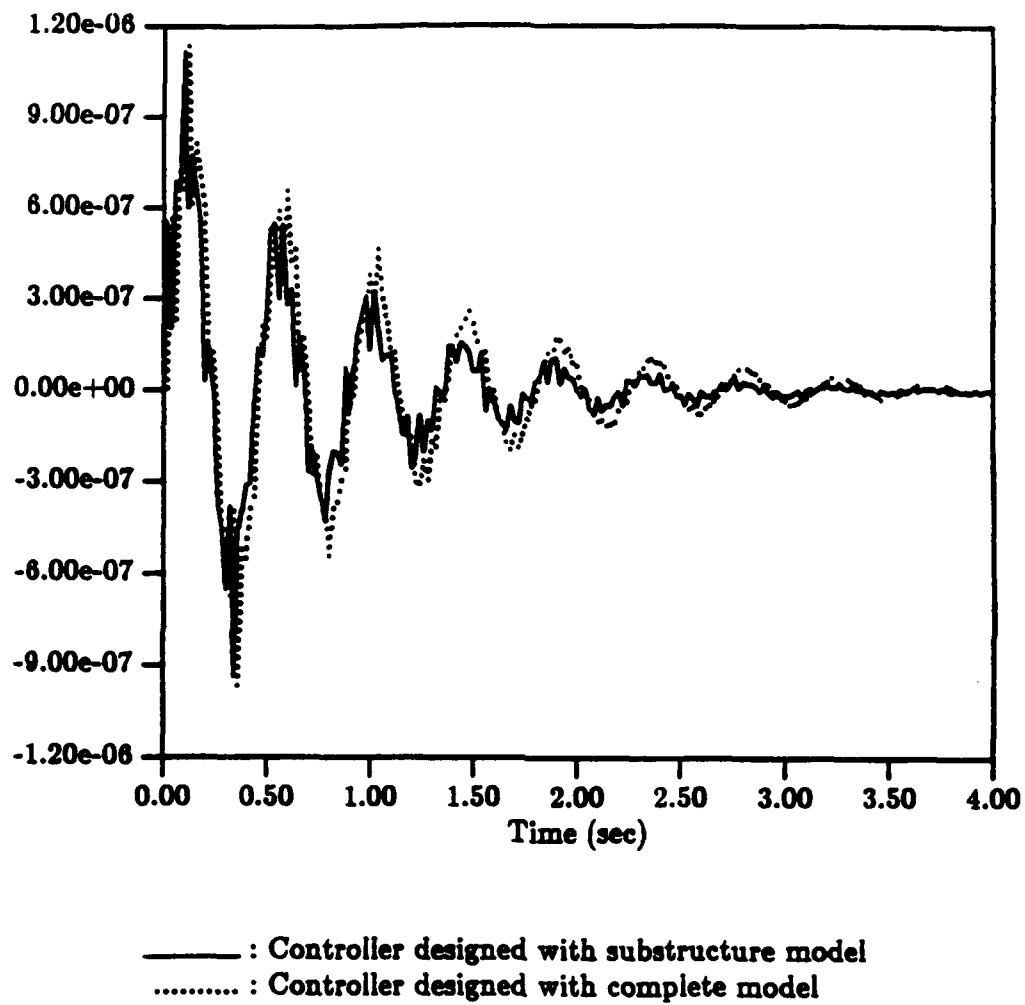
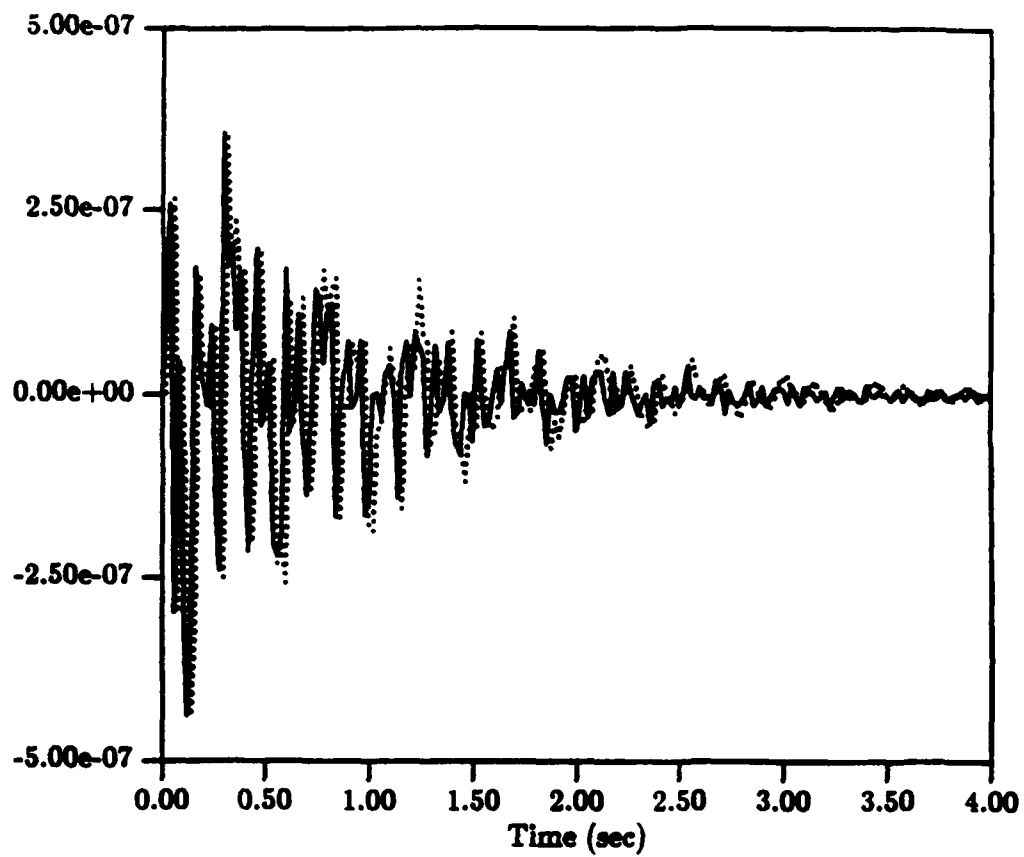


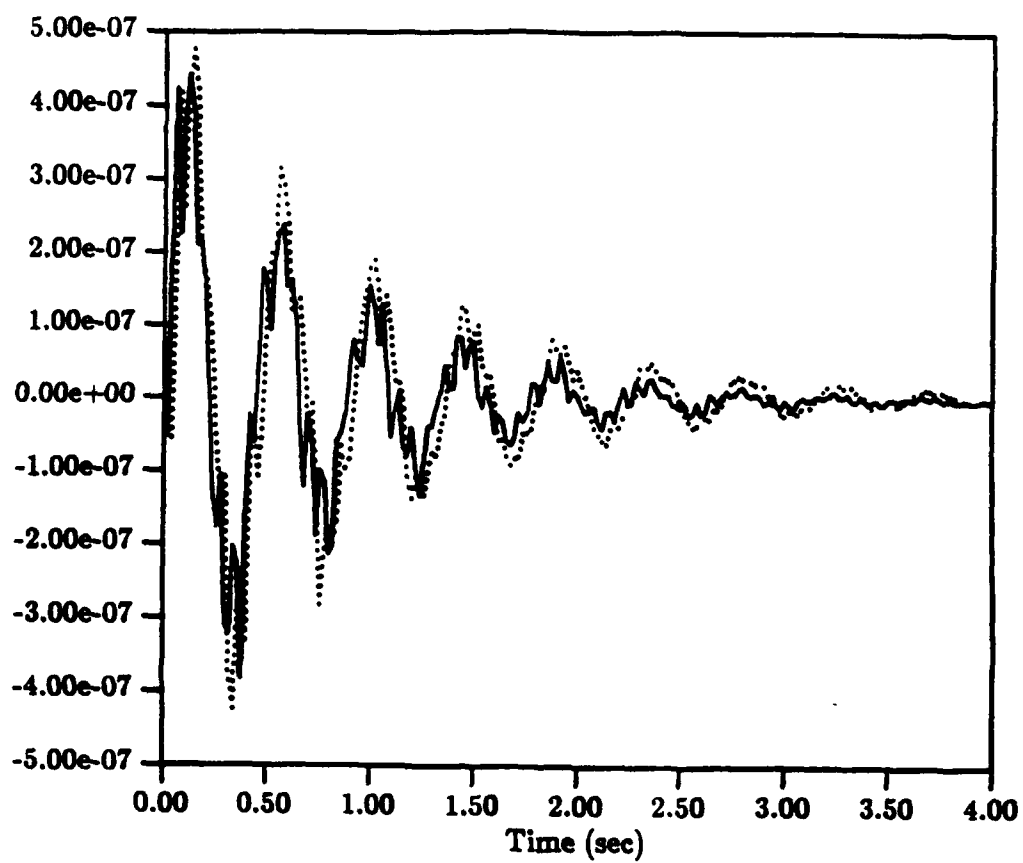
Figure 2.6 Displacement of 2-bay Truss at d.o.f. 2 (impulse at tip)



———— : Controller designed with substructure model  
..... : Controller designed with complete model

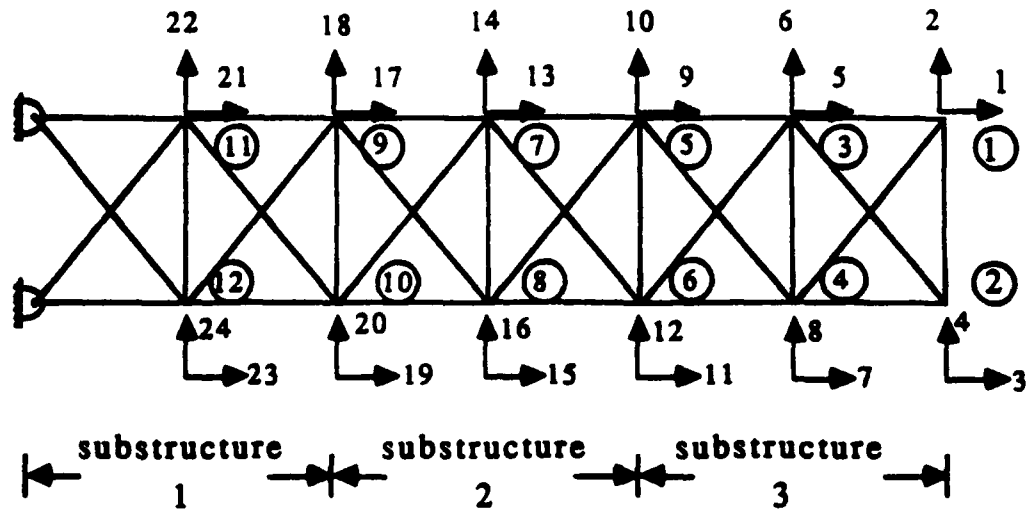
Figure 2.7 Displacement of 2-bay Truss at d.o.f. 5 (impulse at tip)





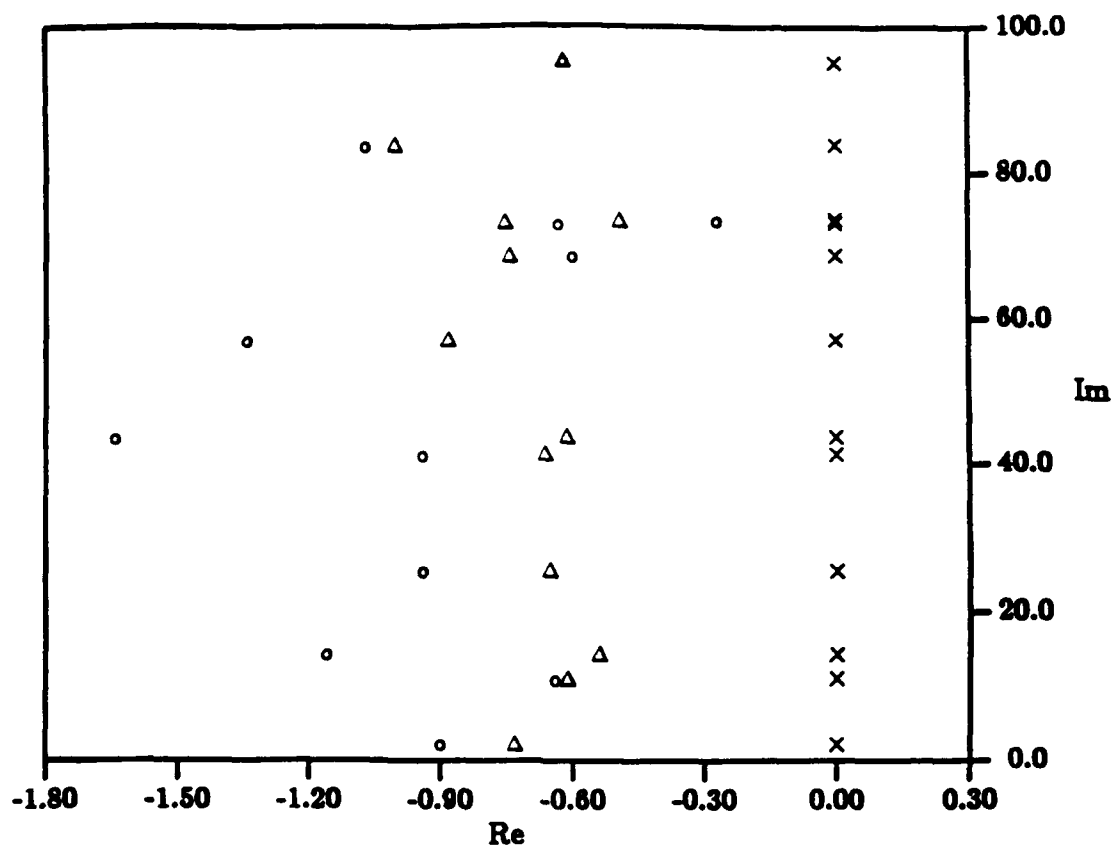
—— : Controller designed with substructure model  
..... : Controller designed with complete model

Figure 2.8 Displacement of 2-bay Truss at d.o.f. 6 (impulse at tip)



**Note:** Actuators and sensors are colocated at d.o.f. 1,2,5,6,17, and 18  
 Numbers with circle represent the node numbers  
 Substructure 1 contains (1,2,3,4,5,6) excluding element (5,6)  
 Substructure 2 contains (5,6,7,8,9,10) excluding element (9,10)  
 Substructure 3 contains (9,10,11,12,13,14)

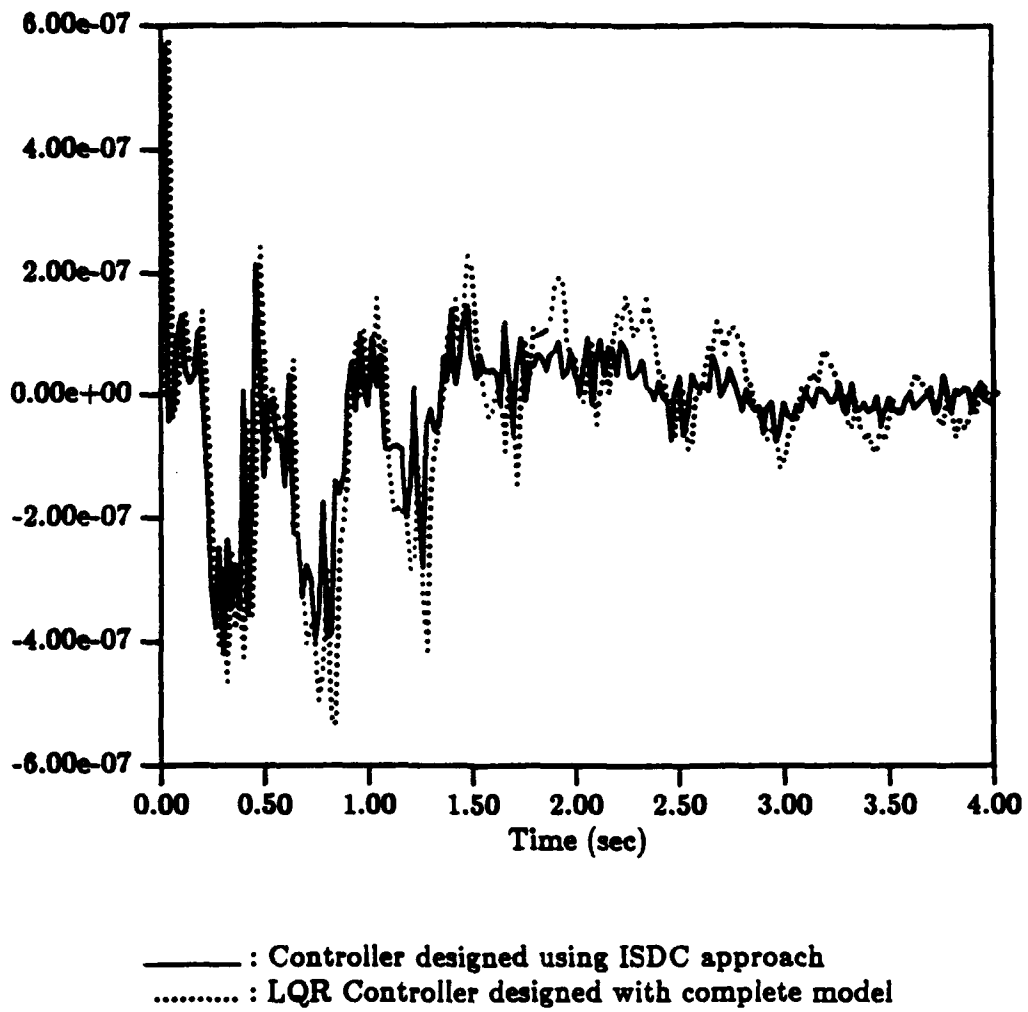
**Figure 2.9 Six-Bay Truss**



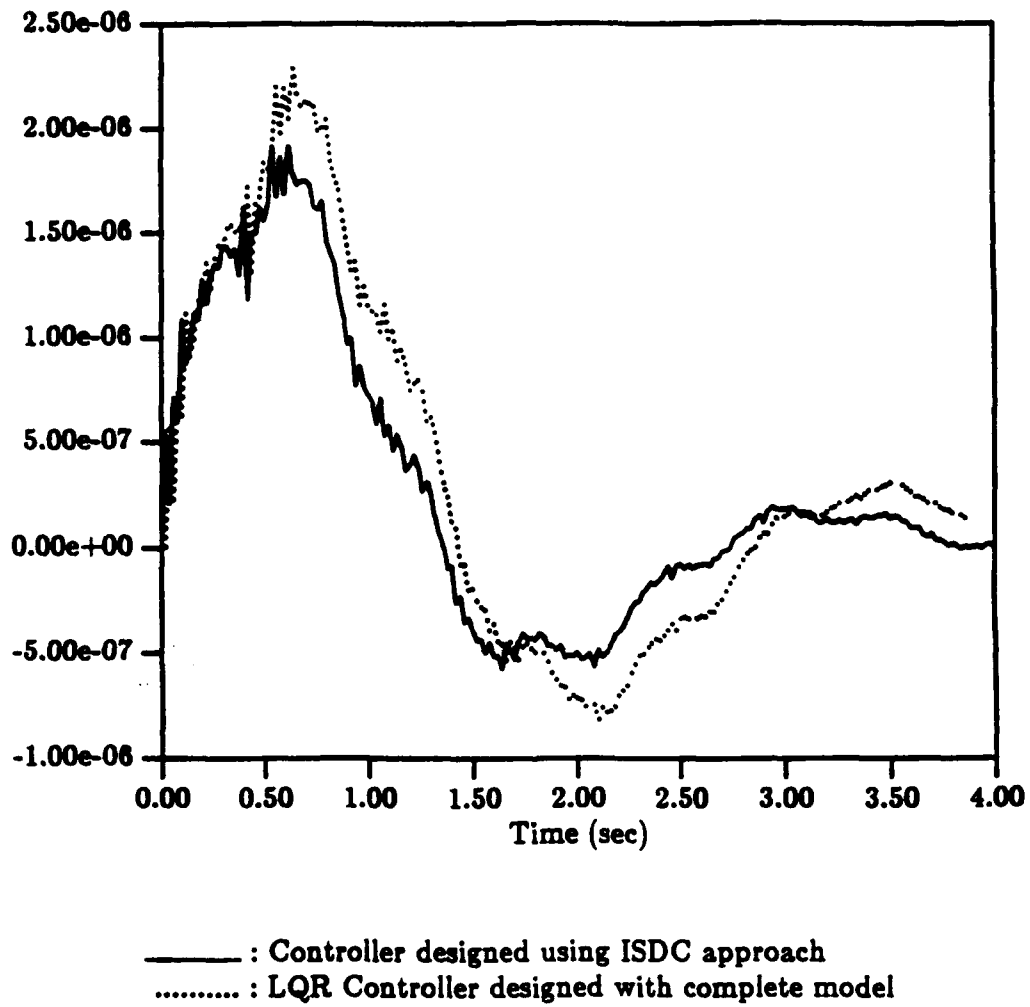
Legend :

- × : Eigenvalues of open-loop structure
- Δ : Eigenvalues of closed-loop system with controller designed with complete model
- : Eigenvalues of closed-loop system with controller designed with substructure model

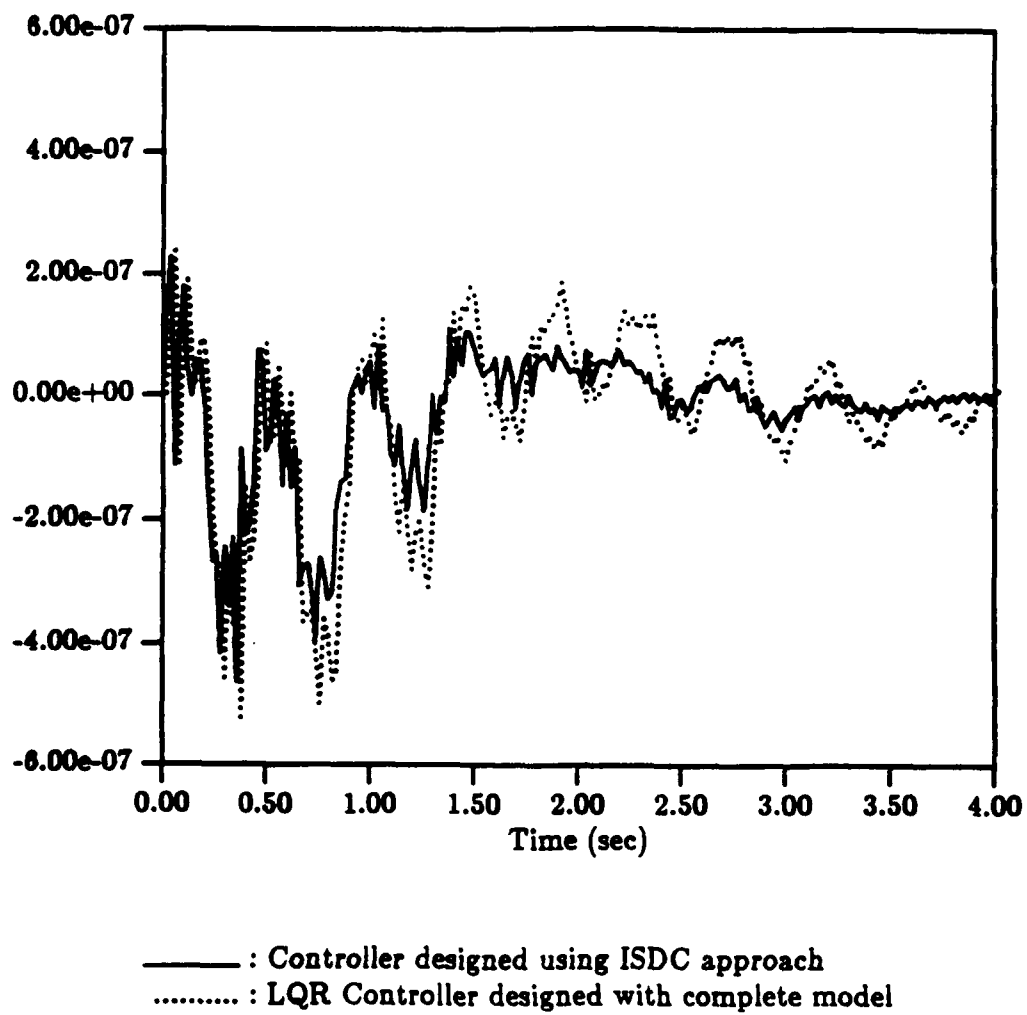
Figure 2.10 Eigenvalues of 6-bay Truss (first 12 pairs)



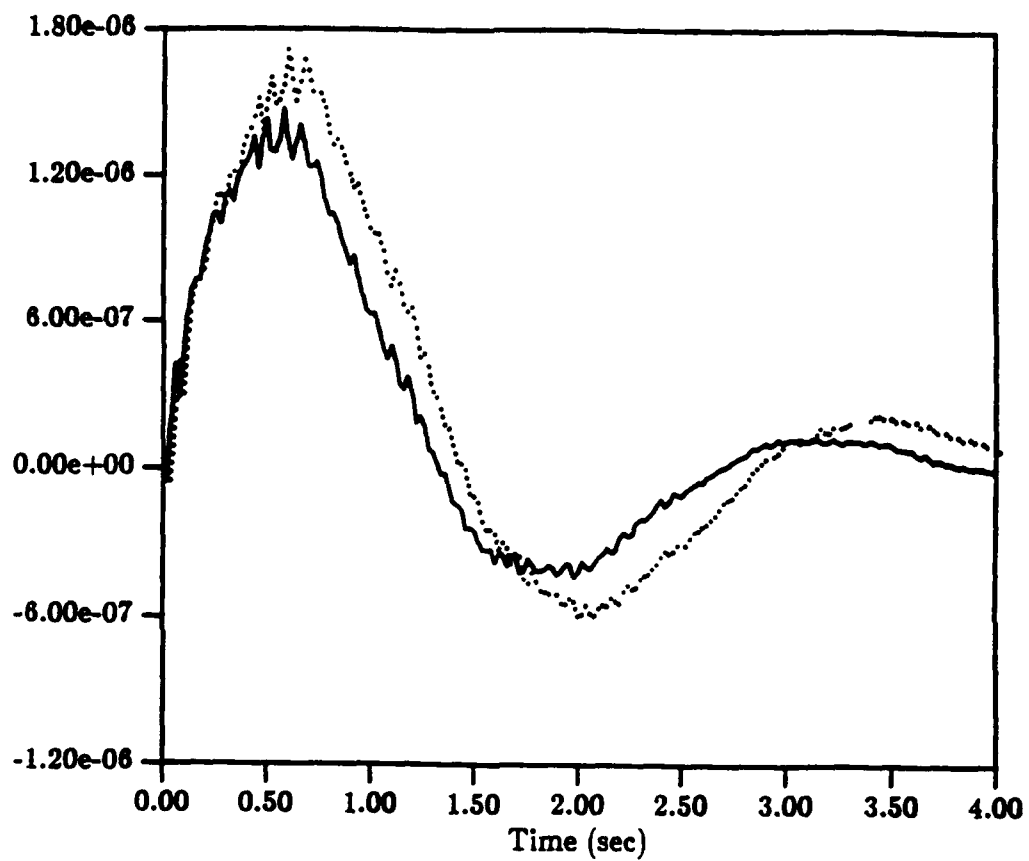
**Figure 2.11** Displacement of 6-bay Truss at d.o.f. 1 (impulse at tip)



**Figure 2.12** Displacement of 6-bay Truss at d.o.f. 2 (impulse at tip)

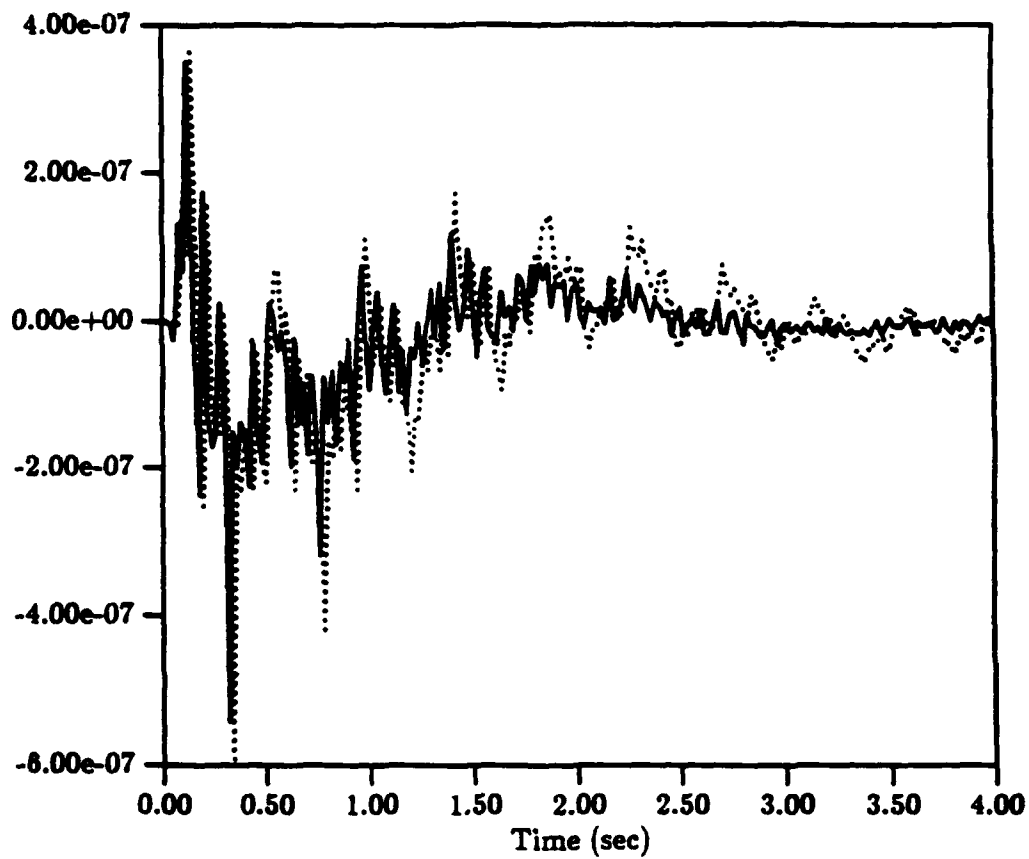


**Figure 2.13** Displacement of 6-bay Truss at d.o.f. 5 (impulse at tip)



— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

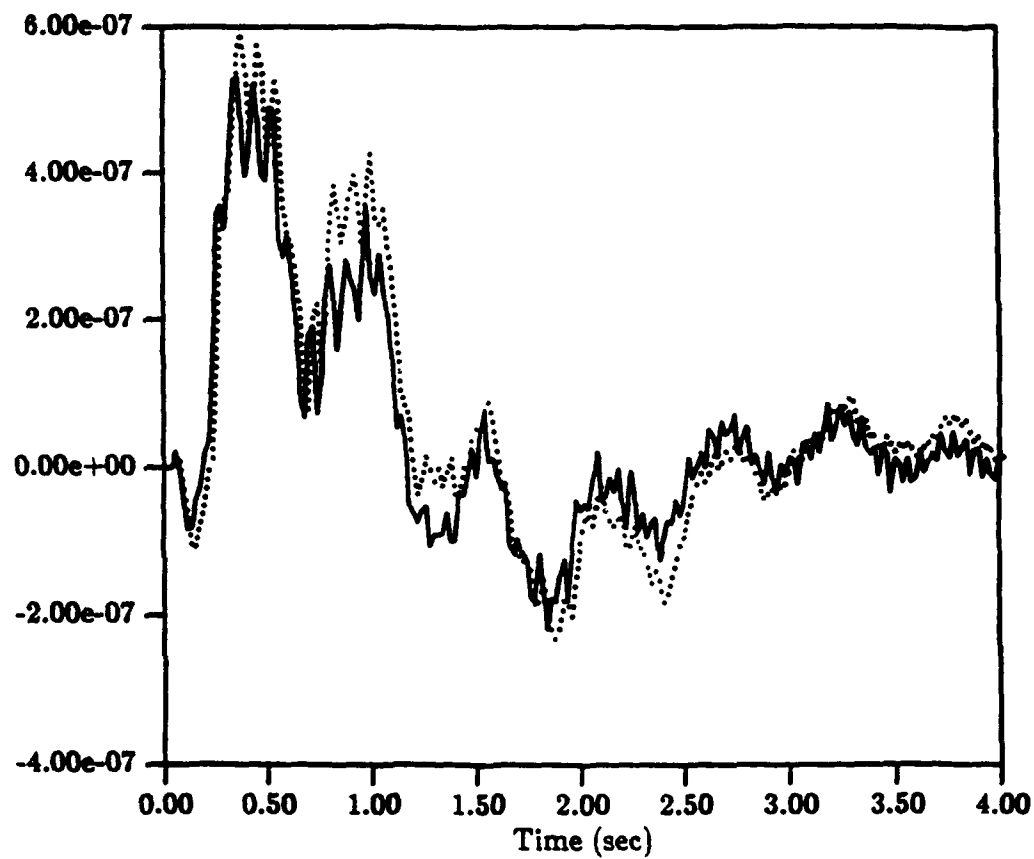
Figure 2.14 Displacement of 6-bay Truss at d.o.f. 6 (impulse at tip)



— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

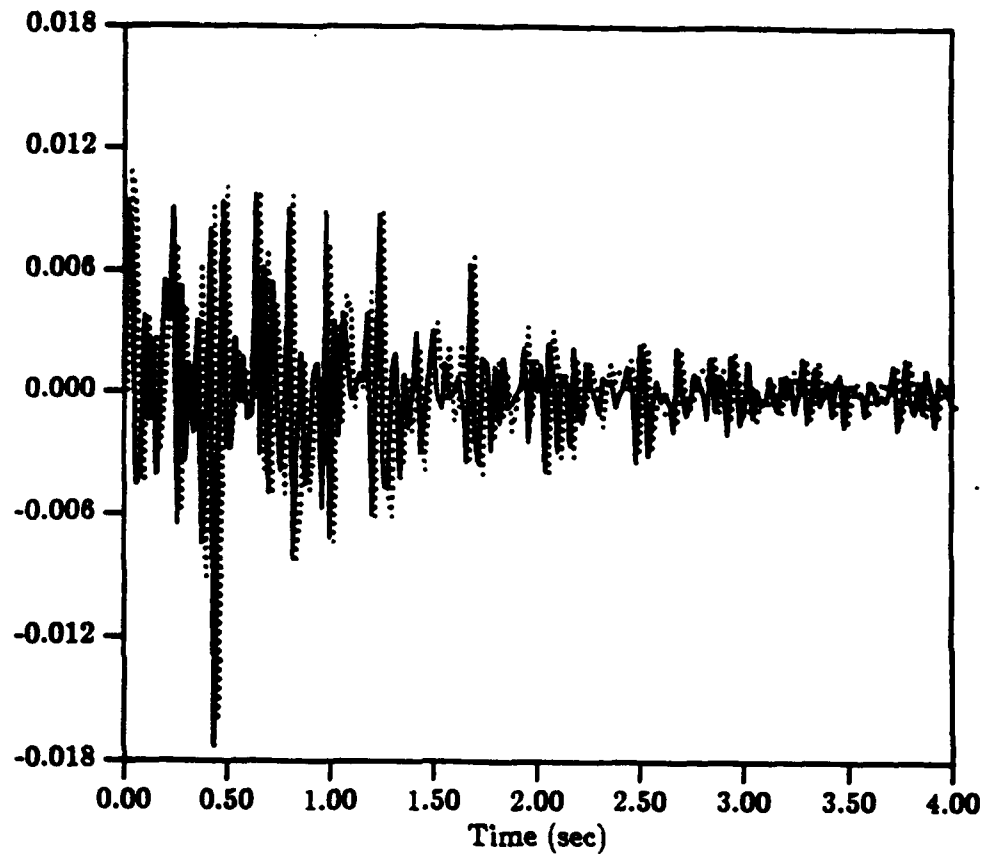
**Figure 2.15** Displacement of 6-bay Truss at d.o.f. 17 (impulse at tip)





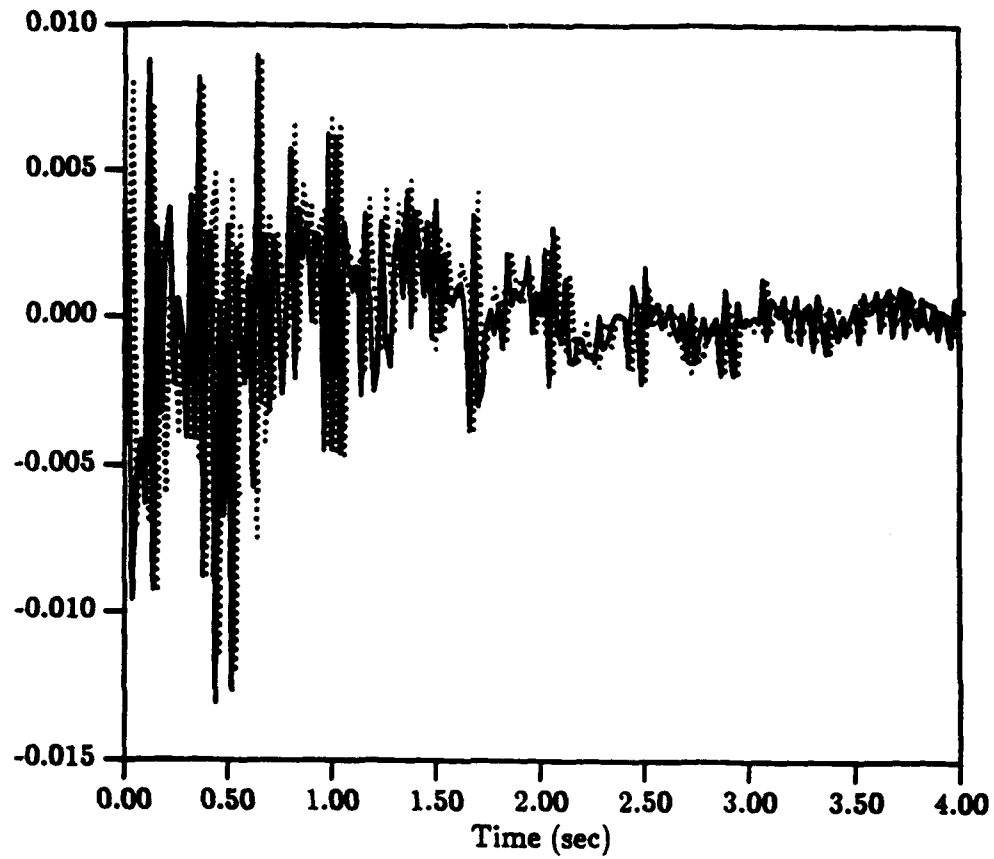
—— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

**Figure 2.16** Displacement of 6-bay Truss at d.o.f. 18 (impulse at tip)



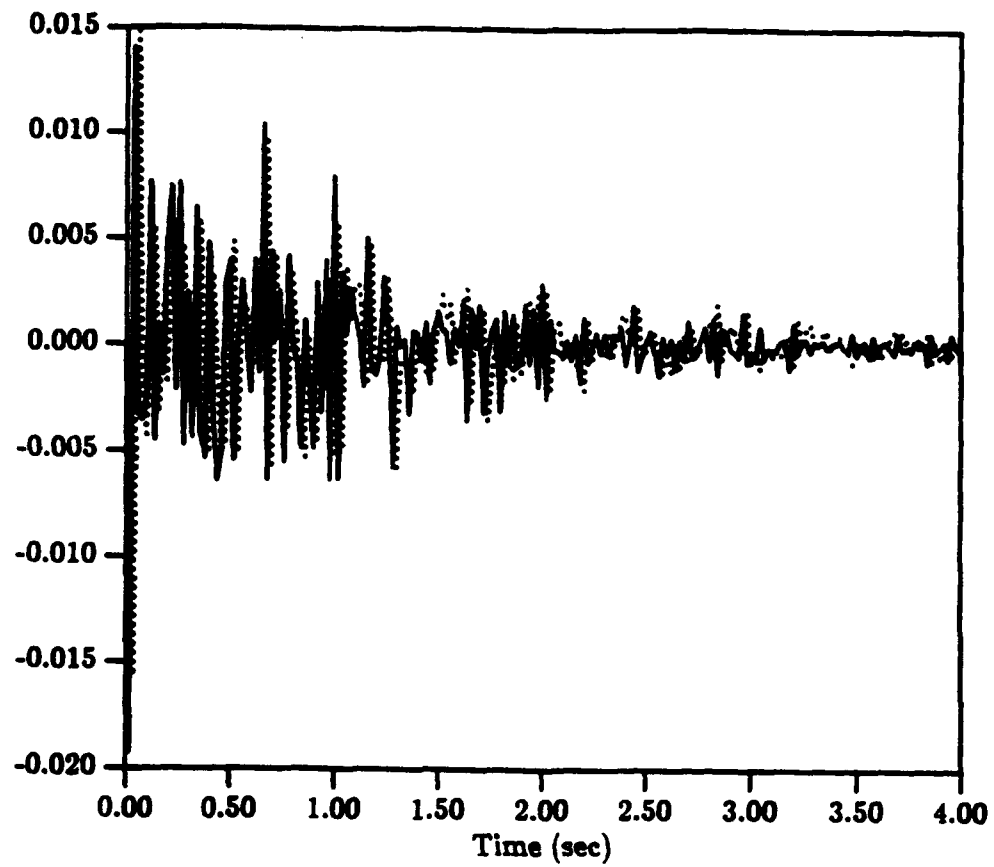
—— : Controller designed using ISDC approach  
..... : LQR Controller designed with complete model

**Figure 2.17** Input Force of Actuator 1 (d.o.f. 1) for 6-bay Truss (impulse)



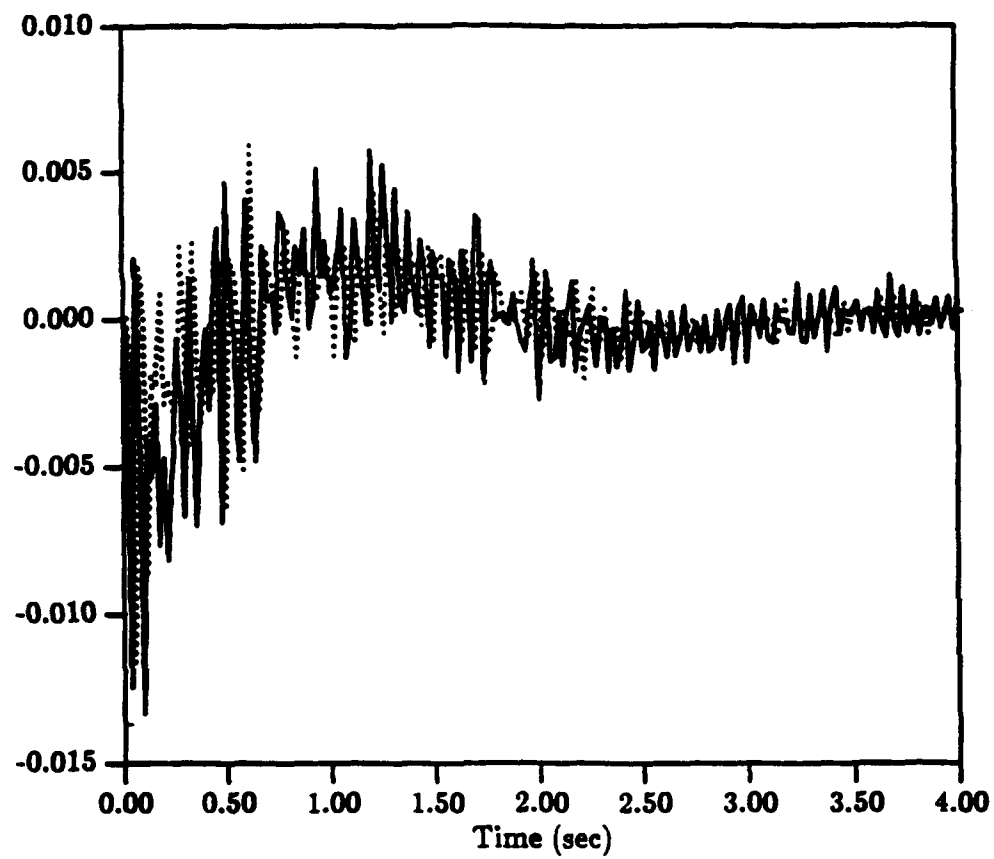
—— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

Figure 2.18 Input Force of Actuator 2 (d.o.f. 2) for 6-bay Truss (impulse)



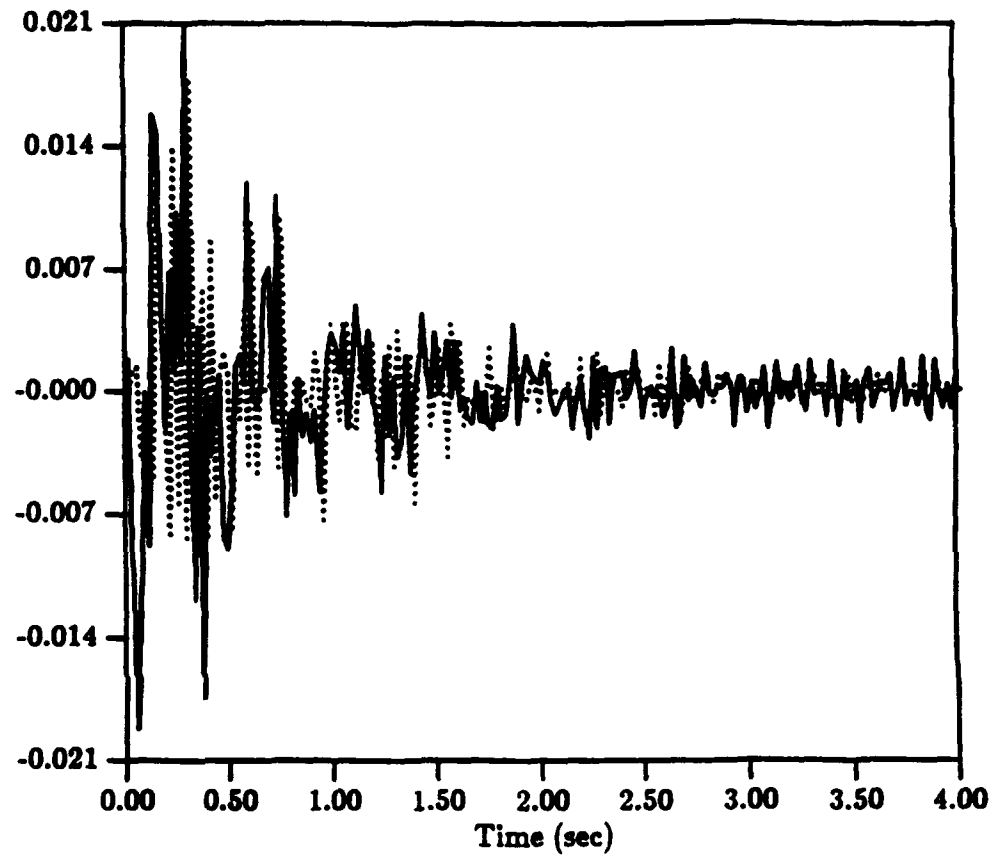
—— : Controller designed using ISDC approach  
..... : LQR Controller designed with complete model

**Figure 2.19** Input Force of Actuator 3 (d.o.f. 5) for 6-bay Truss (impulse)



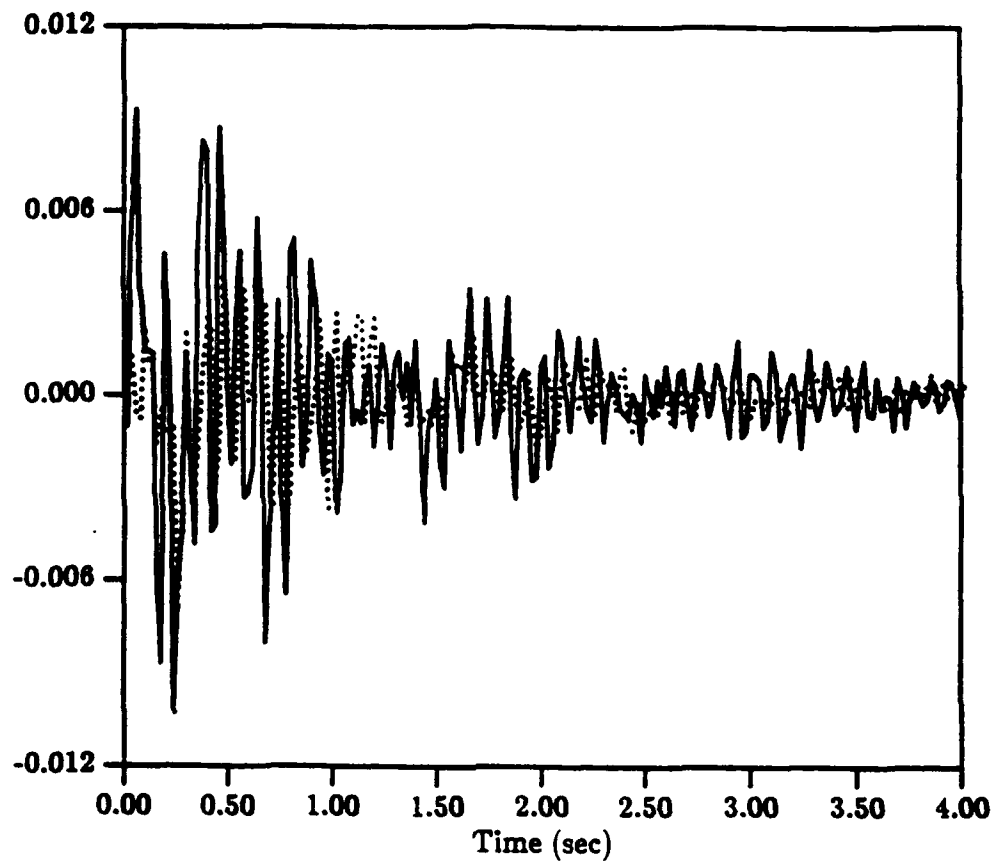
—— : Controller designed using ISDC approach  
..... : LQR Controller designed with complete model

**Figure 2.20** Input Force of Actuator 4 (d.o.f. 8) for 6-bay Truss (impulse)



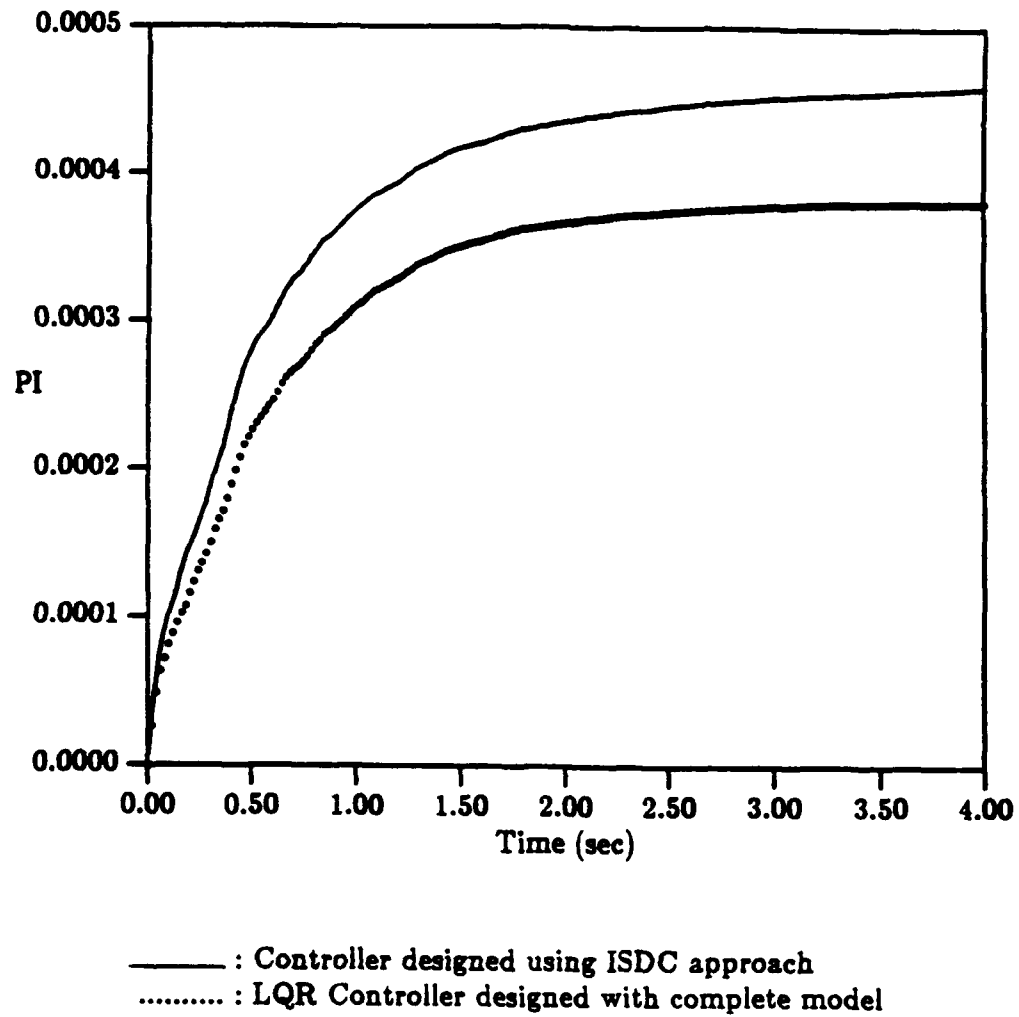
— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

**Figure 2.21** Input Force of Actuator 5 (d.o.f. 17) for 6-bay Truss (impulse)



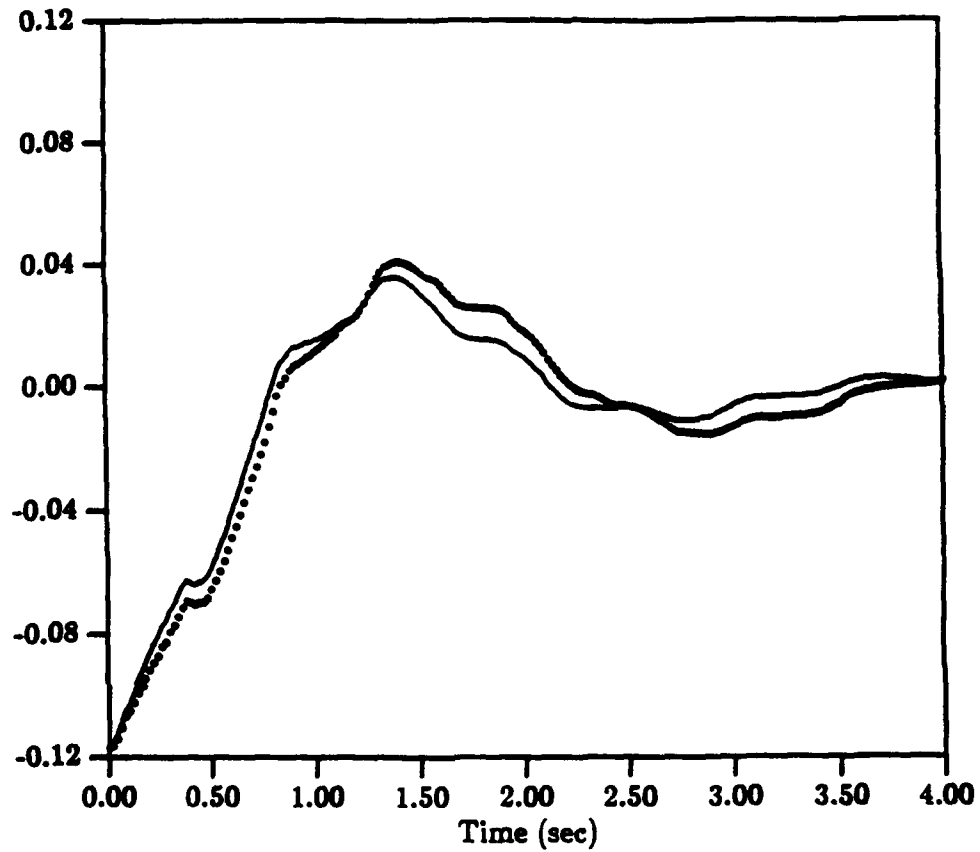
—— : Controller designed using ISDC approach  
..... : LQR Controller designed with complete model

**Figure 2.22** Input Force of Actuator 6 (d.o.f. 18) for 6-bay Truss (impulse)



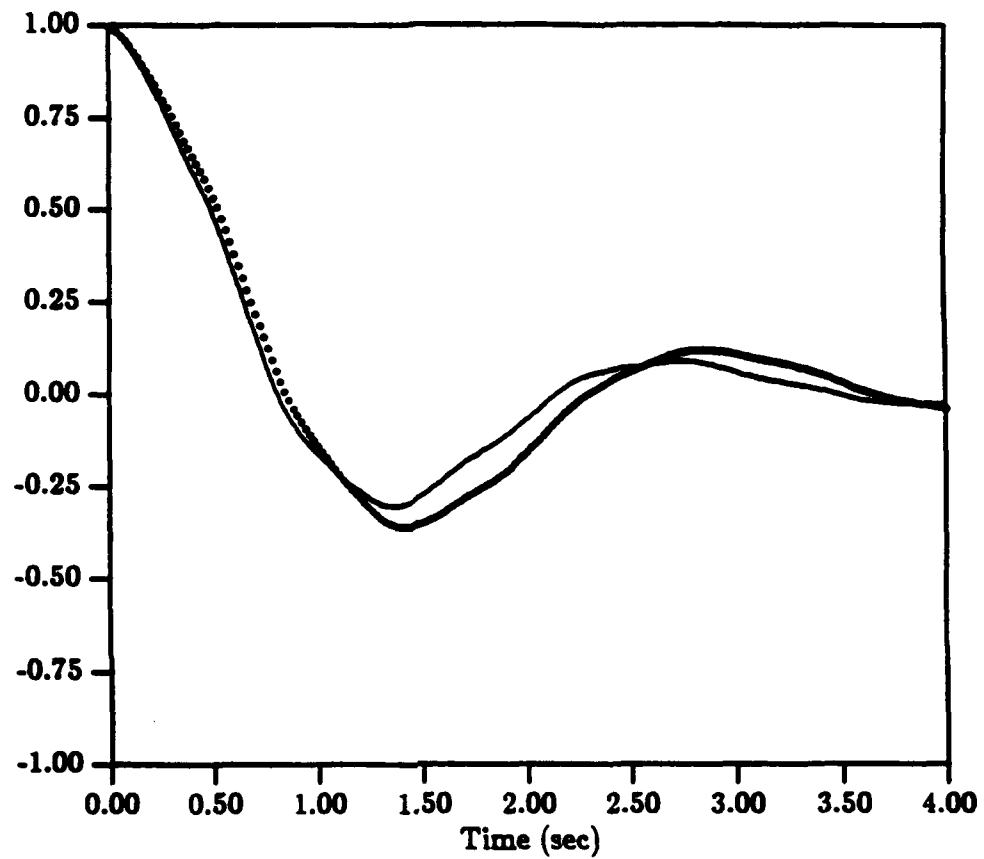
**Figure 2.23** Performance Index vs Time for 6-bay Truss (impulse)





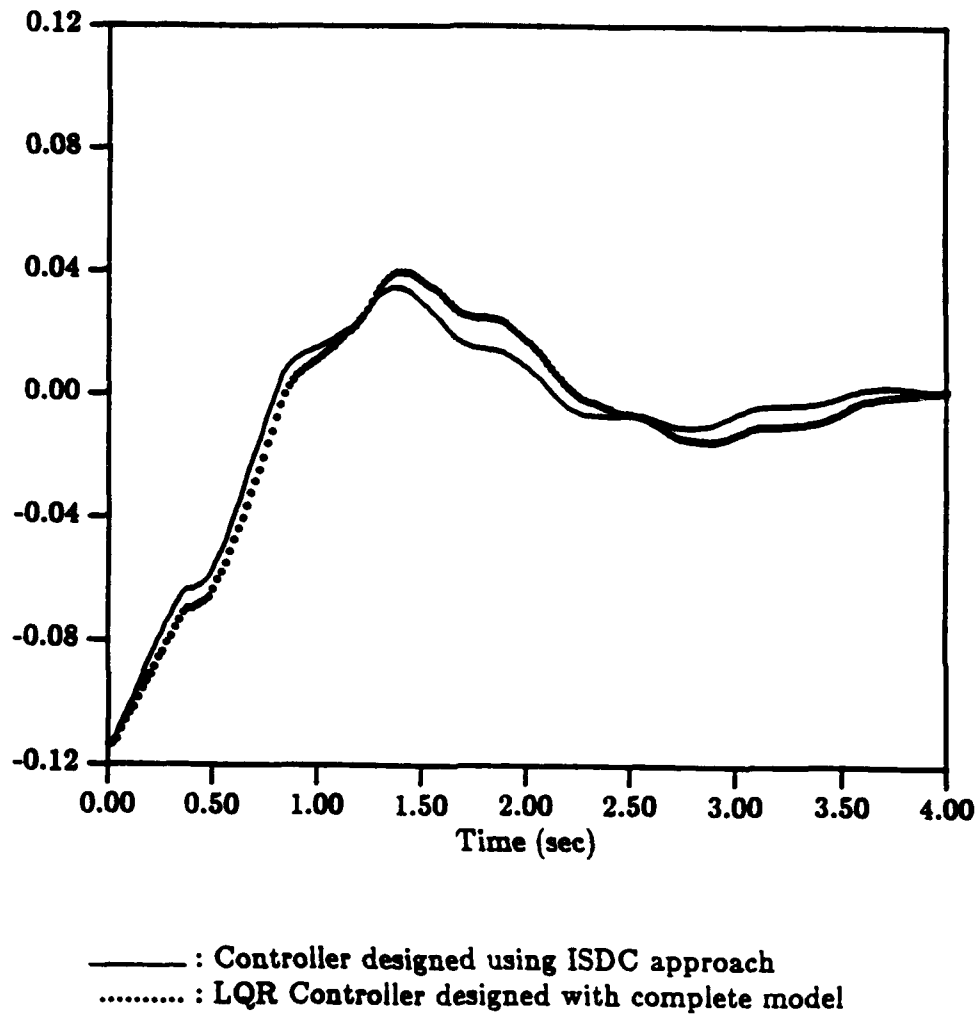
—— : Controller designed using ISDC approach  
..... : LQR Controller designed with complete model

**Figure 2.24** Displacement of 6-bay Truss at d.o.f. 1

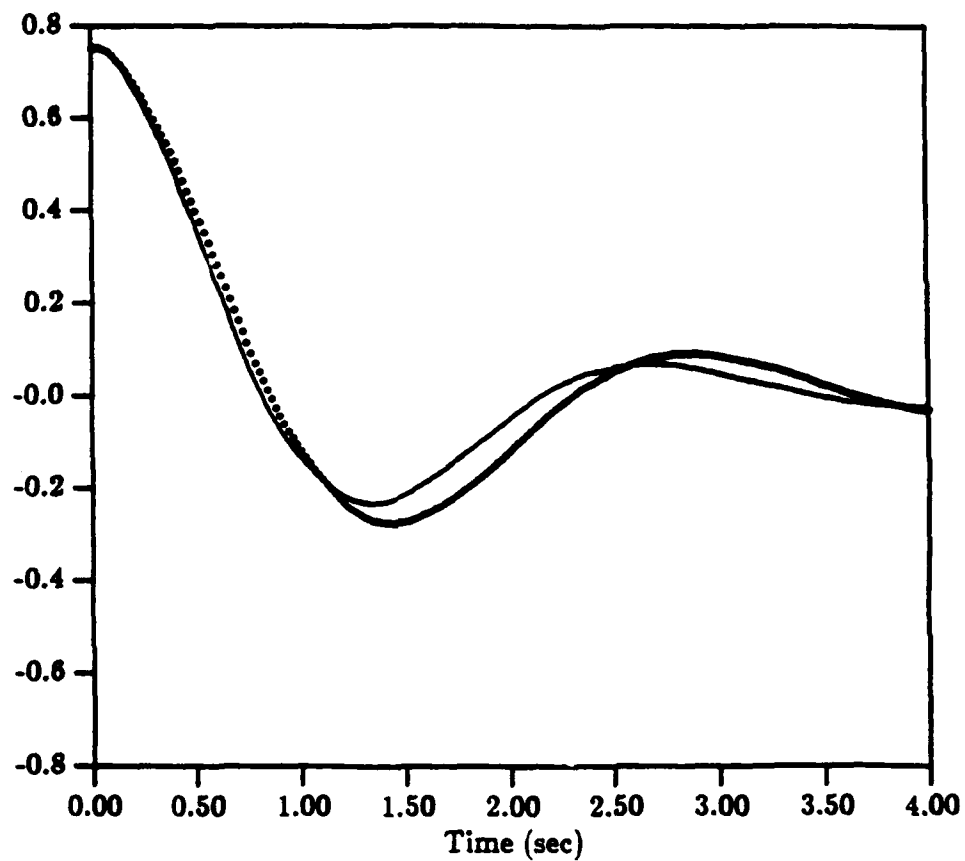


—— : Controller designed using ISDC approach  
..... : LQR Controller designed with complete model

**Figure 2.25** Displacement of 6-bay Truss at d.o.f. 2

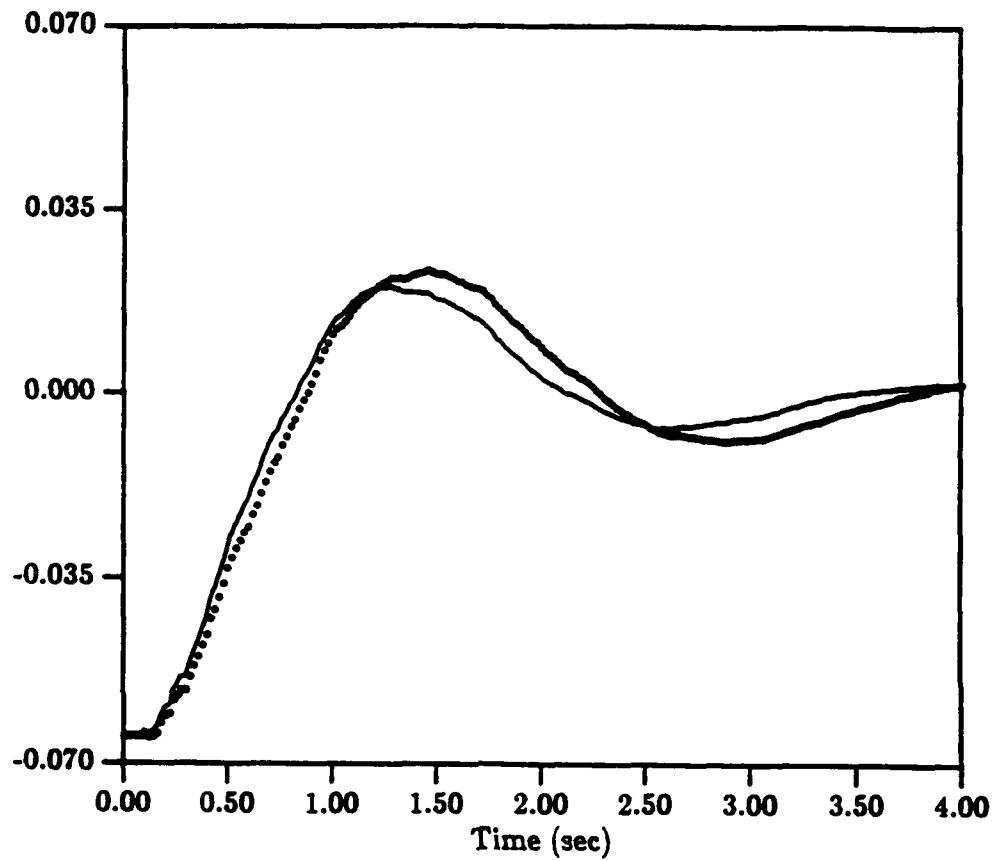


**Figure 2.26** Displacement of 6-bay Truss at d.o.f. 5



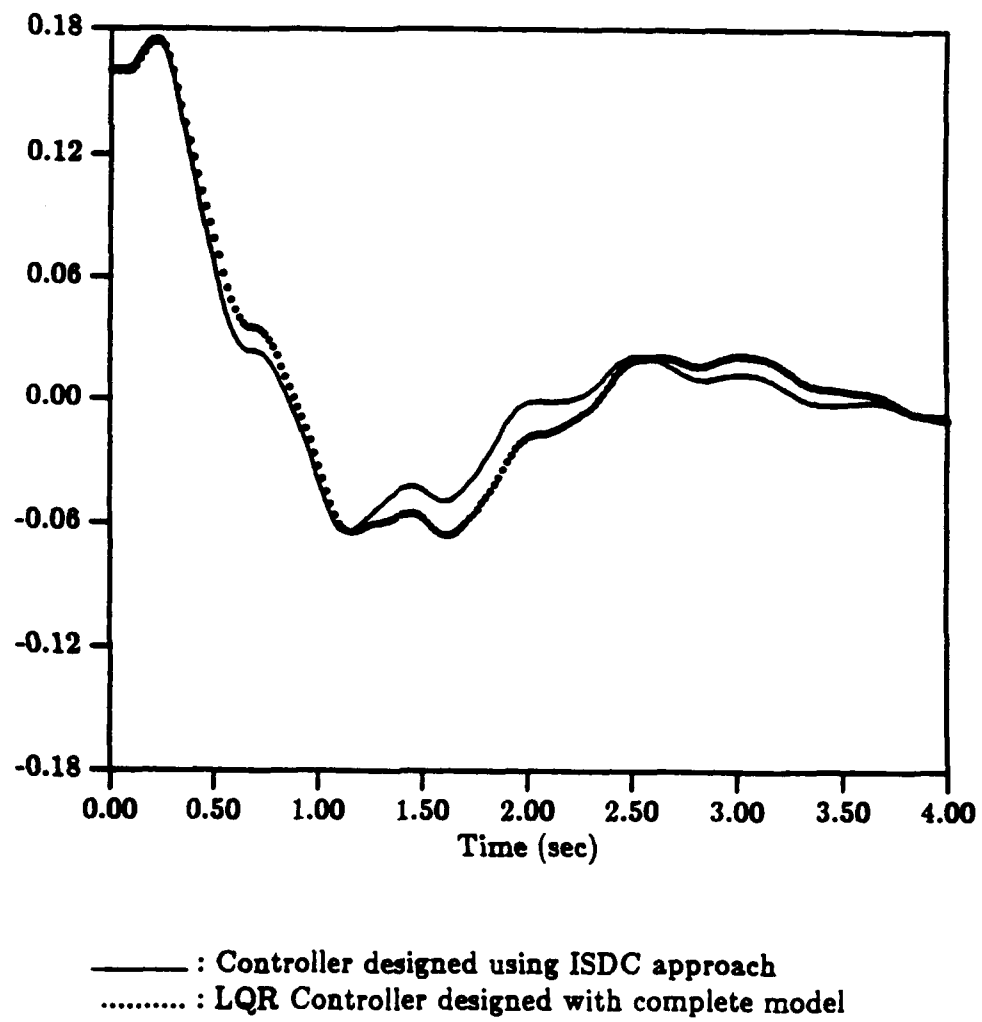
— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

**Figure 2.27** Displacement of 6-bay Truss at d.o.f. 6

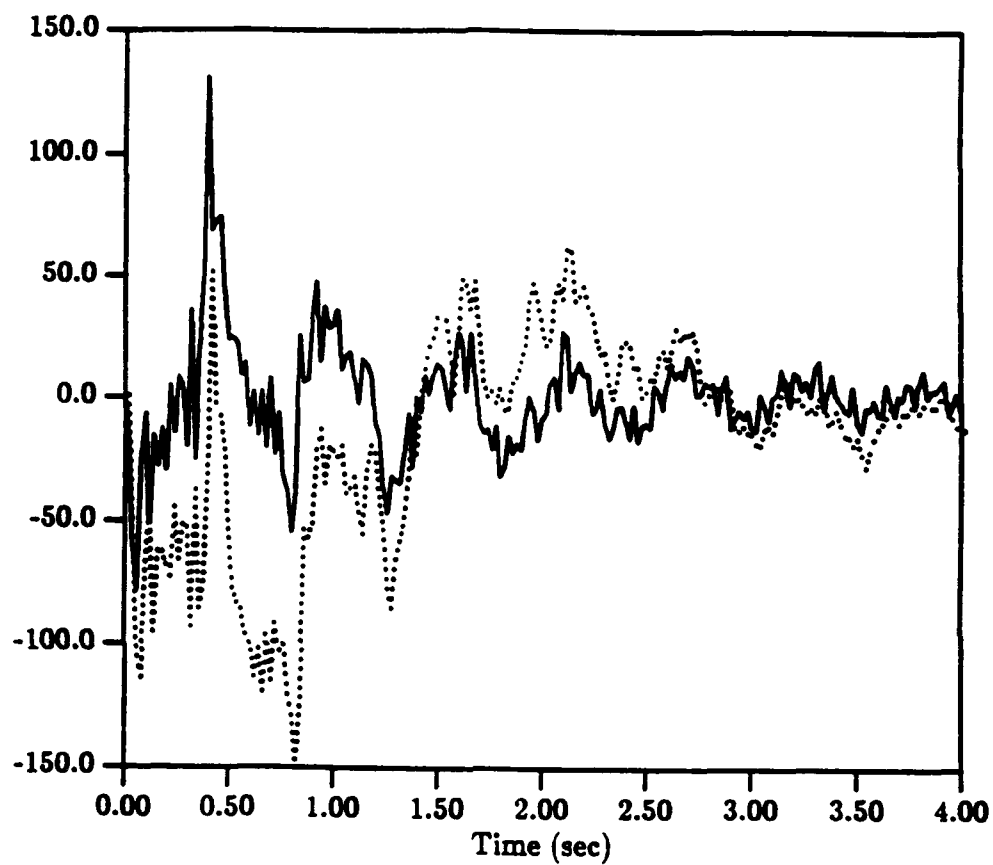


—— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

**Figure 2.28** Displacement of 6-bay Truss at d.o.f. 17

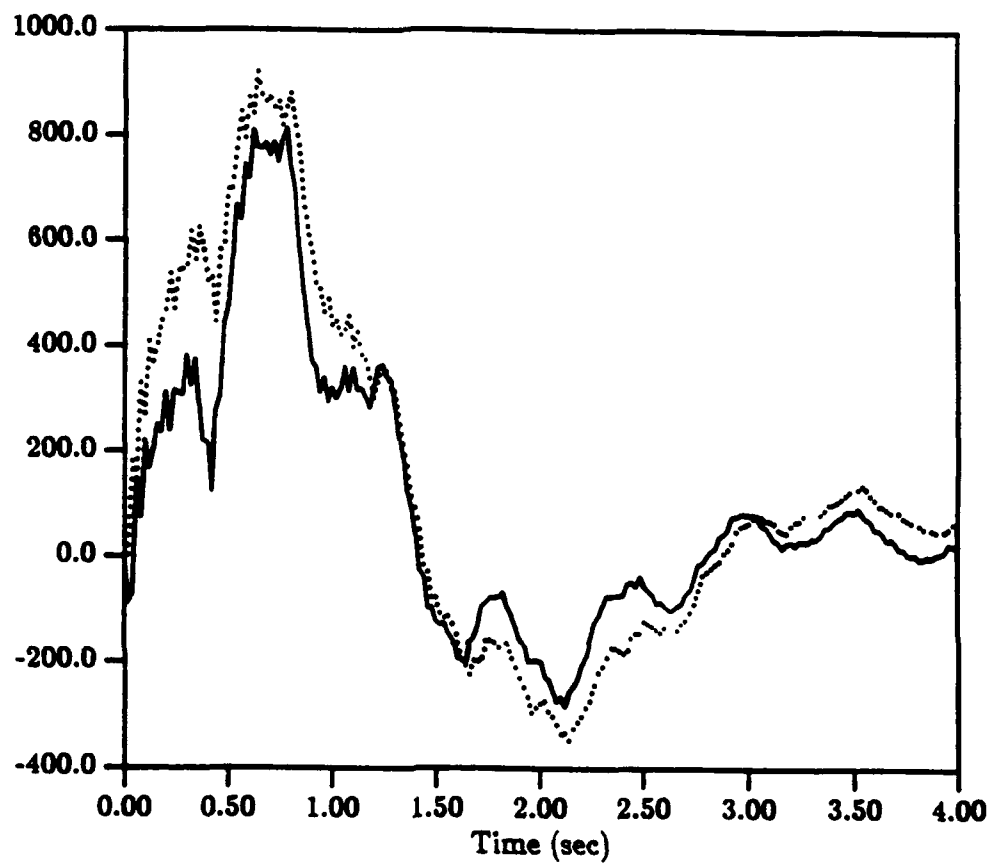


**Figure 2.29** Displacement of 6-bay Truss at d.o.f. 18



— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

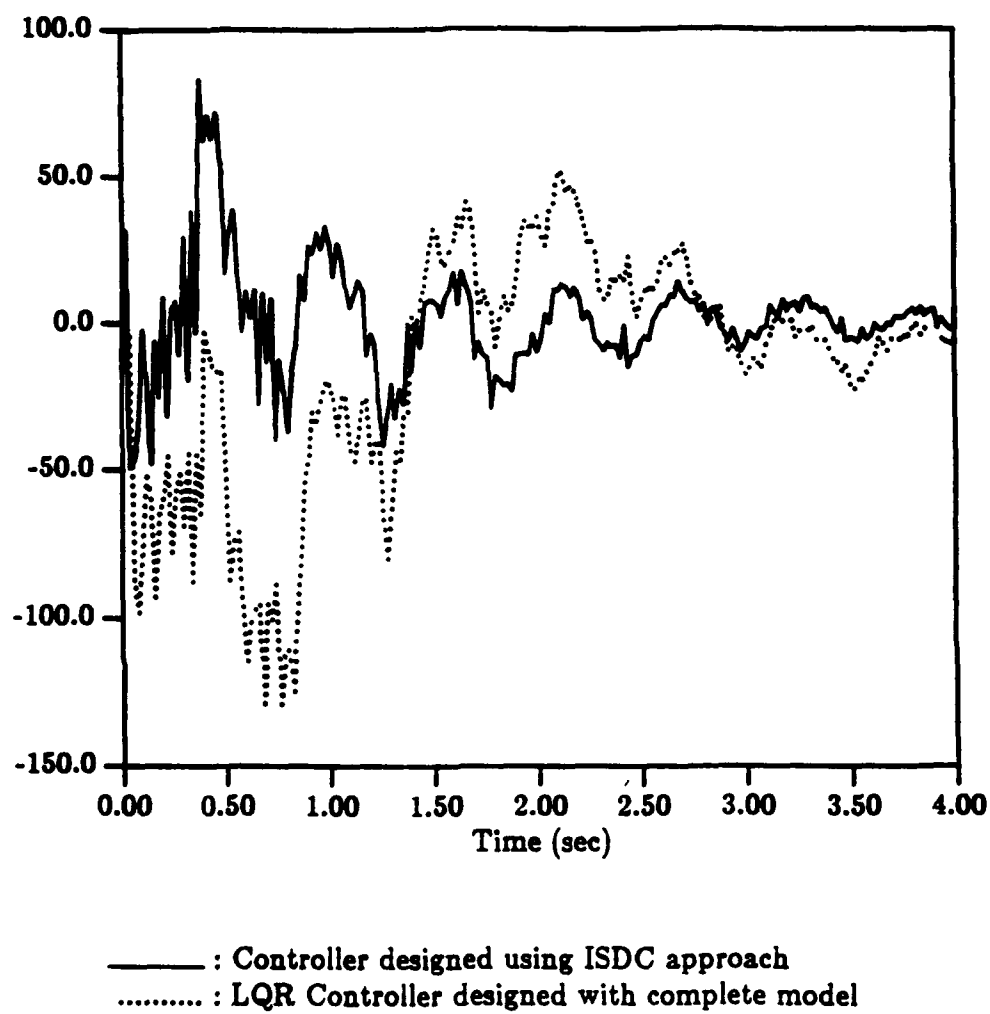
**Figure 2.30** Input force of Actuator 1 (d.o.f. 1) for 6-bay Truss



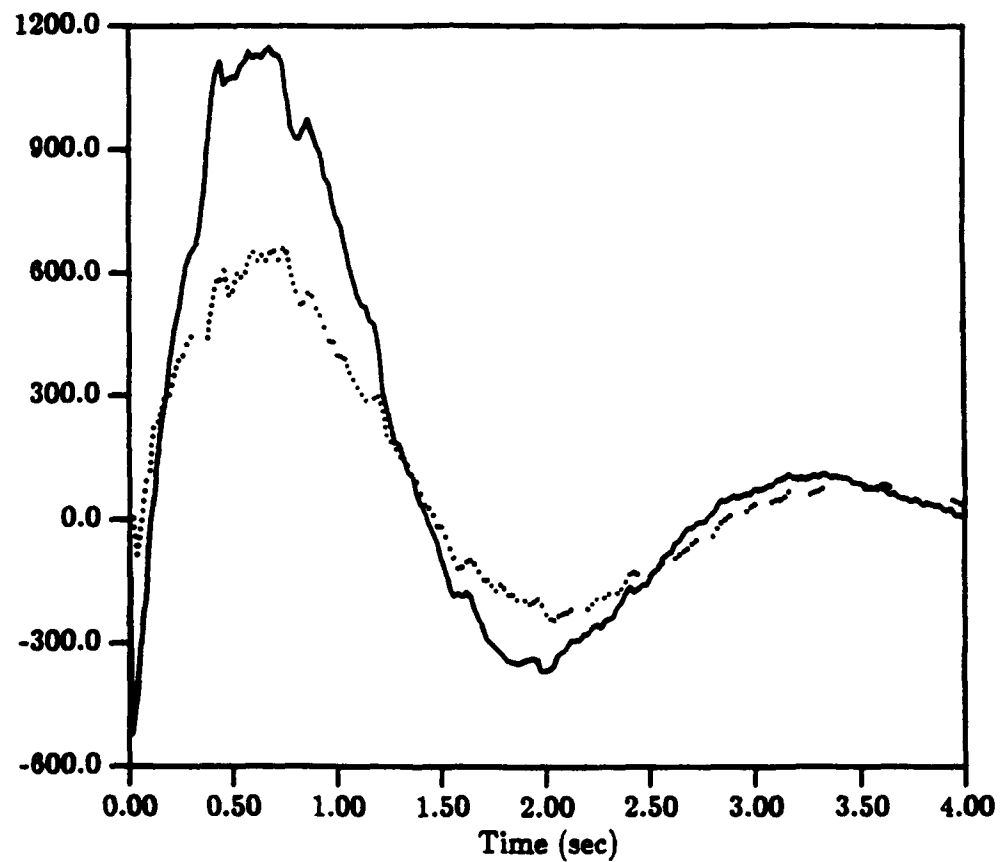
—— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

**Figure 2.31** Input Force of Actuator 2 (d.o.f. 2) for 6-bay Truss



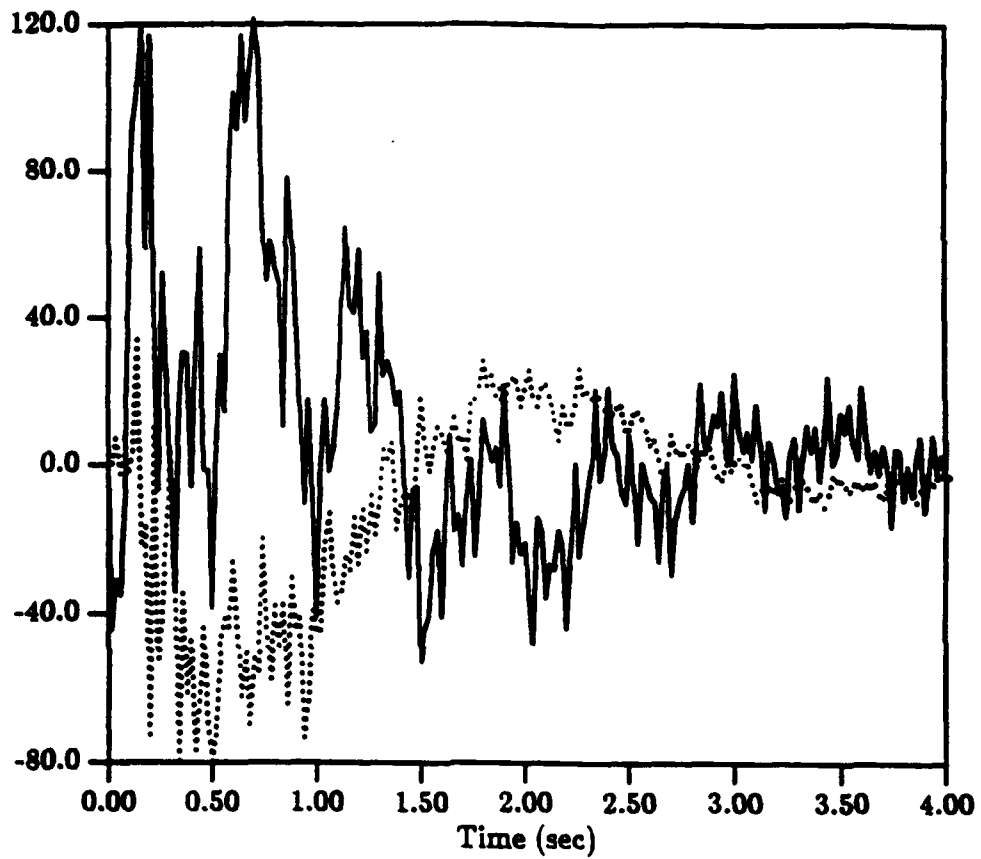


**Figure 2.32** Input Force of Actuator 3 (d.o.f. 5) for 6-bay Truss



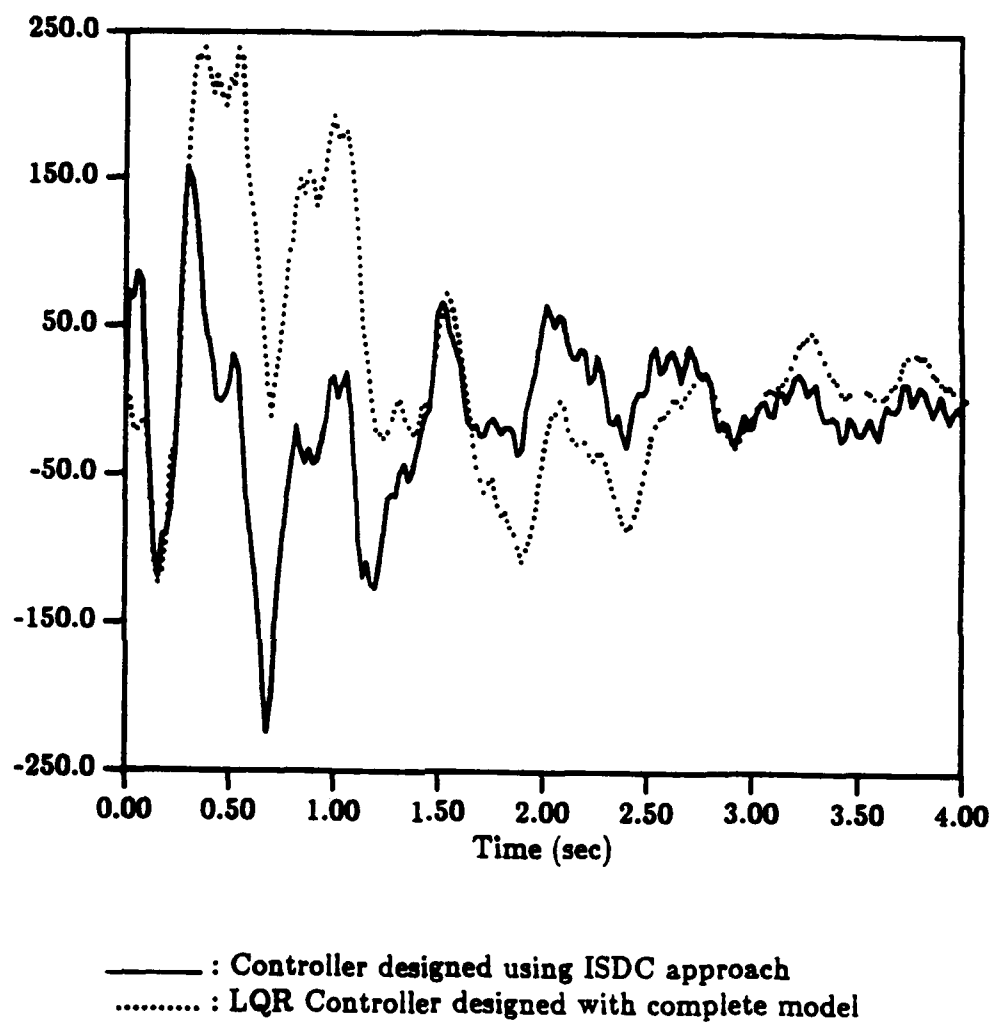
—— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

**Figure 2.33** Input Force of Actuator 4 (d.o.f. 8) for 6-bay Truss

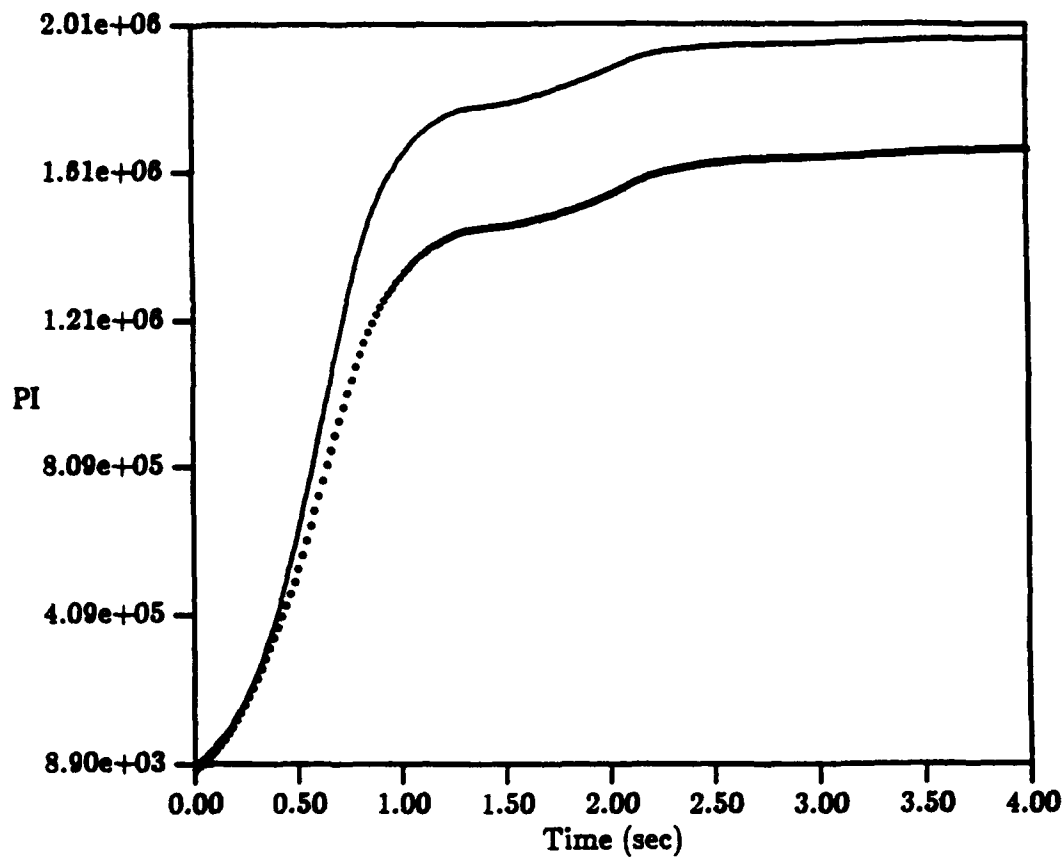


———— : Controller designed using ISDC approach  
..... : LQR Controller designed with complete model

**Figure 2.34** Input Force of Actuator 5 (d.o.f. 17) for 6-bay Truss

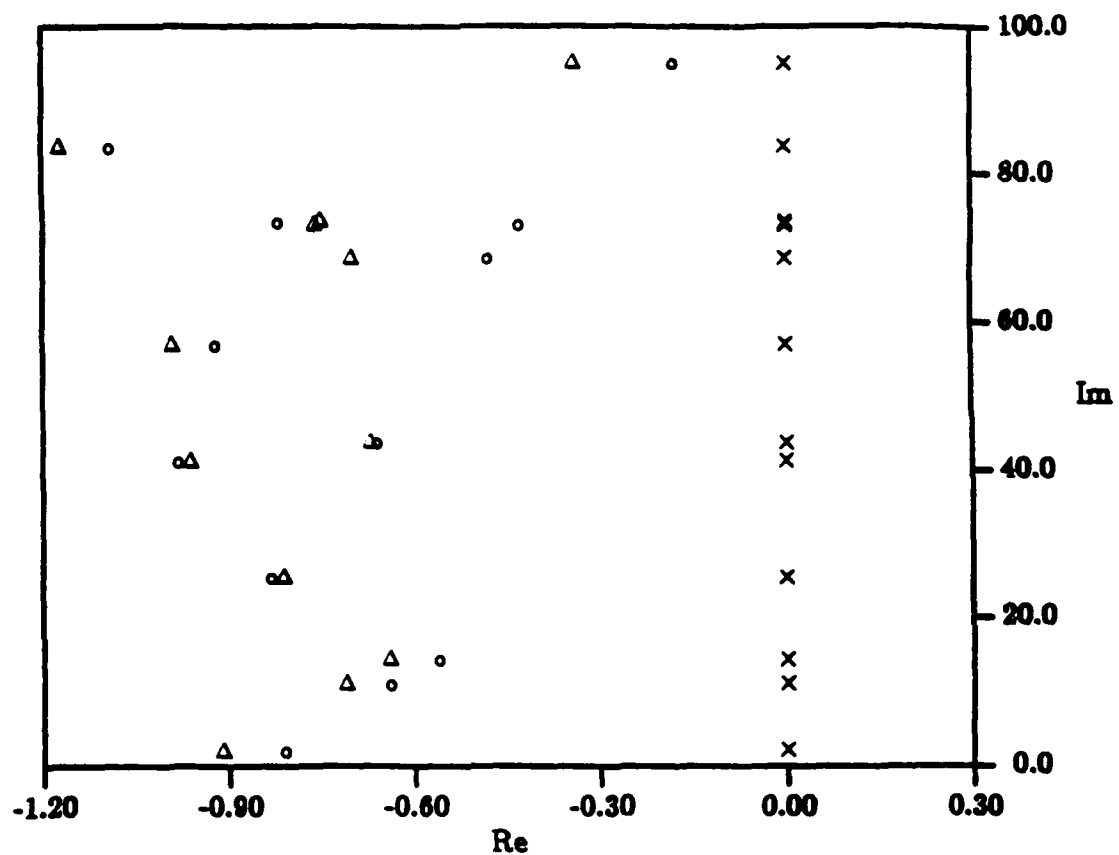


**Figure 2.35** Input Force of Actuator 6 (d.o.f. 18) for 6-bay Truss



—— : Controller designed using ISDC approach  
 ..... : LQR Controller designed with complete model

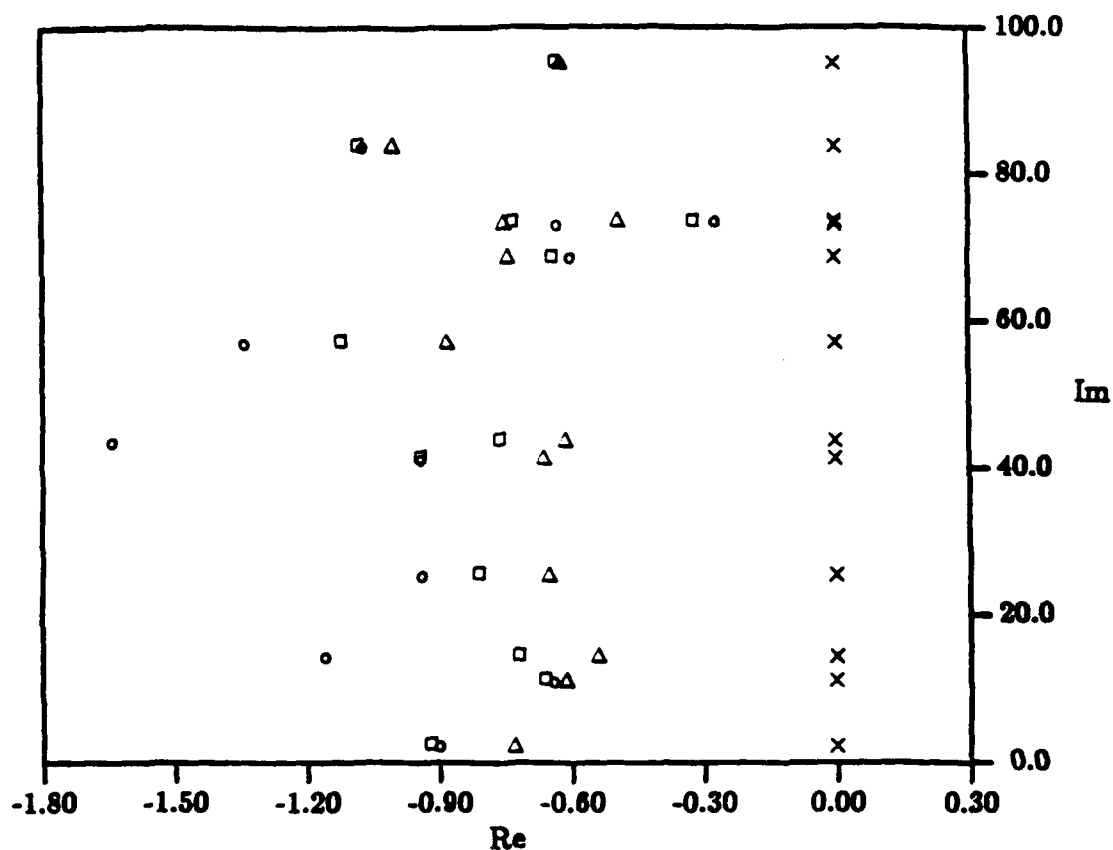
**Figure 2.36** Performance Index vs Time for 6-bay Truss



Legend :

- × : Eigenvalues of open-loop structure
- Δ : Eigenvalues of closed-loop system with controller designed with complete model
- : Eigenvalues of closed-loop system with controller designed with substructure model

Figure 2.37 Eigenvalues of 6-bay Truss (first 12 pairs)



Legend :

- × : Eigenvalues of open-loop structure
- Δ : Eigenvalues of closed-loop system with controller designed with complete model
- : Eigenvalues of closed-loop system with ISDC controller 1 (static transf.)
- : Eigenvalues of closed-loop system with ISDC controller 2 (truncated model transf.)

Figure 2.38 Eigenvalues of 6-bay Truss with Different Controllers (first 12 pairs)

## CHAPTER 3

### DUAL PASSIVE/ACTIVE CONTROL

#### 3.1 Unified Passive Damper Design

Usually, discrete passive dampers are added to flexible structures at specific locations to increase the stability of the whole structure as well as to improve the robustness of the closed-loop system. In this section, an approach is developed for passive damper design. The basic idea is to design, first a feedback controller for the original structure, and then a set of passive dampers to approximate the behavior of the feedback controller. The purpose of this passive damper design procedure is to use the well developed control design techniques for designing a near-optimal set of dampers in the sense of minimizing the performance index of the closed-loop system. The performance index is defined as the sum of the vibration energy and the input energy of actuators. The input energy of the actuators can be considered to be proportional to the weight of the damping devices in the unified passive damper design.

For flexible structures, the equations of motion can be written in the form of Eqs.(2.1) and (2.2). If the optimal linear quadratic regulator is used for the controller design, the control gain can be obtained by solving the algebraic Riccati equation (2.6) and Eq.(2.5). Partitioning the Riccati solution matrix  $P$  into four  $n \times n$  submatrices (where  $n$  denotes the dimension of the displacement vector  $v$ ) as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \quad (3.1)$$

and substituting Eqs.(2.5) and (3.1) into Eq.(2.2) leads to the closed-loop system

$$\dot{x} = A_{cl}x \quad (3.2)$$

where

$$A_{cl} = \begin{bmatrix} 0 & I \\ -M^{-1}DR^{-1}D^T M^{-T}P_{12} - M^{-1}K & -M^{-1}DR^{-1}D^T M^{-T}P_{22} - M^{-1}C \end{bmatrix} \quad (3.3)$$

Equation (3.2) can be rewritten as



$$M\ddot{x} + (C+\Delta C)\dot{x} + (K+\Delta K)x = 0 \quad (3.4)$$

where

$$\Delta K = DR^{-1}D^T M^{-T} P_{12} \quad (3.5)$$

and

$$\Delta C = DR^{-1}D^T M^{-T} P_{22} \quad (3.6)$$

The changes in C and K, namely,  $\Delta C$  and  $\Delta K$ , brought by the active controller can also be partly achieved using a set of discrete dampers and springs. When m dampers ( $c_i$ ) and p spring ( $k_i$ ) at several specified locations are used, the damping and stiffness matrices of the system are modified by  $\Delta\bar{C}$  and  $\Delta\bar{K}$  respectively. The values of  $c_i$  and  $k_i$  can be determined by using the least squares method such that the differences between  $\Delta\bar{C}$  ( $\Delta\bar{K}$ ) and  $\Delta C$  ( $\Delta K$ ) are minimized. The least squares problem is formulated and solved as follows:

1. Write  $\Delta\bar{C}$ ,  $\Delta C$ ,  $\Delta\bar{K}$  and  $\Delta K$  in vector form  $\bar{C1}$ ,  $C1$ ,  $\bar{K1}$  and  $K1$  (e.g.  $\bar{C1}_l = \Delta\bar{C}_{ij}$  and  $l=(i-1)*n+j$ ).
2. Write  $\bar{C1}$  and  $\bar{K1}$  into a constant matrix multiplying an unknown vector such that

$$\bar{C1} = A \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} \quad (3.7)$$

$$\bar{K1} = B \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_p \end{bmatrix} \quad (3.8)$$

and A, B are  $n^2*m$  and  $n^2*p$  coefficient matrices.

3.

$$\text{Min}_{c_1 \dots c_m} \|\bar{C1} - C1\| \quad (3.9)$$

$$\text{Min}_{k_1 \dots k_p} \|\bar{K1} - K1\| \quad (3.10)$$

Since  $\Delta\bar{C}$  and  $\Delta C$  are symmetric matrices, we can reduce the order of the problem in Eq. (3.9) to  $m \cdot n(n+1)/2$ , as

$$\text{Min}_{c_1 \dots c_m} \|A' \begin{Bmatrix} c_1 c_2 \dots c_m \end{Bmatrix}^T - C1'\| \quad (3.11)$$

where  $C1'$  is the upper triangular matrix of  $\Delta C$  in vector form, and  $A' \begin{Bmatrix} c_1 c_2 \dots c_m \end{Bmatrix}^T$  is the upper triangular matrix of  $\Delta\bar{C}$  in vector form. Since  $\Delta K$  is a nonsymmetric matrix, Eq.(3.10) cannot be reduced.

4. The Moore-Penrose inverse [14] is used to solve Eqs.(3.11) and (3.10) as

$$\begin{Bmatrix} c_1 c_2 \dots c_m \end{Bmatrix}^T = A'^+ C1' \quad (3.12)$$

$$\begin{Bmatrix} k_1 k_2 \dots k_p \end{Bmatrix}^T = B^+ K1 \quad (3.13)$$

where

$$A'^+ = V\Sigma^+U^T \quad (3.14)$$

$$\Sigma^+ = \text{diag}[\sigma_1^{-1} \sigma_2^{-1} \dots \sigma_r^{-1} 0 \dots 0]$$

$$A' = U\Sigma V^T \text{ (by singular value decomposition)}$$

$$B^+ = V_1 \Sigma_1^+ U_1^T \quad (3.15)$$

$$B = U_1 \Sigma_1 V_1^T \text{ (by singular value decomposition)}$$

5. In this step, some adjustments have to be made. Usually,  $k_i$  and  $c_i$  obtained in step (4) are not all positive semidefinite (i.e., not all realizable); the negative elements should be set equal to zero. If the values of springs are very very small compared to the structural stiffness, the designer can neglect them without much effect.

To get a more general approximation, the constrained optimization techniques can be applied in the presence of constraints on design variables (e.g., size and weight constraints on dampers and stiffnesses). For unconstrained approximation, the least square approach gives the same result as that of a nonlinear optimization approach but the former will save much computational time.

### **3.2 Dual Passive/Active Control**

As we know, passive control is unconditionally stable for a stable open loop system and is robust and reliable. Unfortunately, passive control fails to meet stringent performance requirements which are required in most flexible space structures. On the other hand, active control meets the stringent performance requirements if there are no energy and power limitations and if a perfect system model is available. For practical structures, the model reduction procedures are to be used; also there will be limitations on energy and power. In extreme cases, the spillover problem caused by mode truncation will induce an unstable closed-loop system. Thus, dual passive/active control approach can be used as a compromise approach to meet the performance requirements as well as to make it more tolerant to uncertainties in system design and disturbances. Also, the dual passive/active control design leads to a more reliable control with a smaller production cost.

Gehling, 1986 shows that if passive damping is designed into the system, the amount of damping achievable in a flexible space structure will reduce the requirements of active control design. Savings can be realized in terms of the number of control system components and energy expenditure for vibration control.

The procedure of passive damper design (sec. 3.1) coupled with the interacting substructure decentralized control (Chap. 2) can be described briefly as follows. The first step in dual passive/active control design is to use a well developed control technique to design an active controller and use an approximation process to design a set of preassigned dampers as described in Sec. 3.1. The second step is to modify the original structure with the additional passive dampers and rewrite the system equations of motion. The sensor/actuator location should be preassigned and then the state equation can be written. Finally the interacting substructure decentralized control approach is applied to design the active control system.

### **3.3 Examples**

Four examples are considered to illustrate the procedures outlined in this chapter. The first one deals with the passive damper design using the unified passive damping design (UPD) method on a two-bay truss. The second and third examples deal with the active control using the ISDC method on two-bay and six-bay trusses, respectively. The fourth example deals with the multilevel passive/active control using the UPD and the ISDC methods on a two-bay truss. For original structures, no damping is assumed and hence the open-loop eigenvalues of the

structures are all located on the imaginary axis.

### 3.3.1 Passive Damper Design

A two-bay truss (Fig. 2.3), with prespecified arrangement of damping devices, is used to illustrate the passive damper design methodology. Two types of damping arrangements, as shown in Fig. 3.1, are considered in this example. The weighting matrices  $Q$  and  $R$  are chosen as  $10^3 \cdot I$  and  $I$  ( $I$  is the identity matrix), respectively. Following the UPD procedure, the damping coefficients have been determined as indicated in Table 3.1. The eigenvalue pairs of the modified structures (the original structure with passive dampers) are compared with those of the closed-loop controlled structure with active controller designed by the LQR procedure. The first configuration gives an acceptable approximation to the closed-loop system with active control except for the first three eigenvalue pairs (see Fig. 3.2). The first four eigenvalue pairs of the second configuration are not very close to those of the closed-loop system with active control (Fig. 3.2). This shows that the passive damper design procedure, described in this chapter, can improve the system dynamic response very well in higher frequency modes. However, it is impossible to obtain a passive damper design which gives the exact dynamic response of the actively controlled structure. Hence, the combined passive and active control system is necessary for practical structures with stringent dynamic requirements. The damping coefficients vary with the weighting matrices chosen. The relationship, for the second configuration (Fig. 3.1(b)) of damping arrangement, is presented in Table 3.2. The input weighting matrix  $R$  is the identity matrix and the output weighting matrix  $Q$  is the identity matrix multiplied by a scalar coefficient  $q$ . Tables 3.2 and 3.3 show that the change in damping coefficients, as well as the change in the real part of eigenvalues of the modified structures, is approximately proportional to the square root of the change in the scalar coefficient  $q$ .

### 3.3.2 Dual Passive/Active Control

The unified passive damping design procedure (Fig. 3.2) gives a good approximation to the LQR actively controlled structure except for the first few modes and the interacting substructure decentralized control (ISDC) method gives a better compensation for the first few modes. Hence, a dual passive/active control design for LFS with the first-level control designed by the unified passive damping approach, and the second-level control designed by using ISDC, is expected to be more promising in obtaining a better performance. The two-bay truss shown in Fig. 2.3 is considered to demonstrate the multilevel passive/active control design procedure. The results of the two-bay truss configuration shown in Fig. 3.1(b), obtained with only the passive damper design and weighting matrix  $Q=10^5 \cdot I$ , are used as the first level passive control design in this example. A second level active control design is obtained by using the ISDC procedure with weighting matrices  $Q=10^5 \cdot I$  and  $R=I$  based on the passively damped structure. Two substructures are used and four sensors and actuators are colocated at degrees of freedom 1,2,5, and 6. The eigenvalue pairs of (i) the uncontrolled structure, (ii) the

controlled structure with the controller designed with the complete model, (iii) the controlled structure with controller designed by the ISDC method, and (iv) the structure designed with the dual passive/active control, are shown in Fig. 3.3.

Most of the eigenvalues of controlled structure with UPD/ISDC controller move to the left compared to those of the closed-loop system with ISDC controller. This indicates that the stability margin of the closed-loop system, for the higher frequency modes, is increased by using the dual passive/active control strategy. Although the change is small, the reliability and stability robustness of the system will be significantly improved. The relationship between the change in the eigenvalues of the dual UPD/ISDC controlled structure and that in the output weighting matrix is also illustrated in Table 3.4.

The simulation results of the two-bay truss with unit impulse at tip (in both directions at node 1) for different controllers are presented in Figs. 3.4 to 3.11. The displacement outputs (Figs. 3.4 to 3.7) from the sensors appear to be identical for systems with ISDC controller and dual UPD/ISDC controller. Figures 3.8 to 3.11 show that the input forces of ISDC controller are larger than those of dual UPD/ISDC controller. The time variation of the performance index is presented in Fig. 3.12. The performance index of the dual UPD/ISDC controller is reduced by 10.7% compared to that of LQR controller and by 28.14% compared to that of ISDC controller. This indicates that a great improvement in performance has been achieved by the dual passive/active control design.

### 3.4 Concluding Remarks

A unified passive damper design procedure is developed based on the well developed control design techniques and approximation techniques. The design variables are the weighting matrices  $R$  and  $Q$  for LQR design or control gain for other control design techniques. Appropriate location selection of damping devices is very important for the unified passive damper design procedure. For the case when the weighting matrices,  $R$  and  $Q$ , are assumed to be the identity matrix and the identity matrix multiplied by a scalar  $q$ , the change in the damping coefficients, for a preassigned set of damping devices, varies as the square root of the change in the scalar coefficient  $q$ . The design of constrained viscoelastic layer treatment, a kind of continuous damper, can also be achieved by this method using optimization techniques. The dual passive/active control strategy, using unified passive damper design and interacting substructure decentralized control approaches, is applied for a two-bay truss. The result shows that the passive control significantly improves the reliability of the closed-loop system and reduces the active control energy consumption under the same performance requirements.

**Table 3.1 Damping Coefficients of the UPD for Two-Bay Truss ( $Q=10^3 \cdot I$ )**

Damping Coeff. (lb*s/in)		
Configuration	I	II
$c_1$	31.6271	15.8116
$c_2$	21.0915	21.0915
$c_3$	31.6271	15.8116
$c_4$	31.6267	15.8151
$c_5$	21.0871	10.5457
$c_6$	31.6267	15.8151

**Table 3.2 Damping Coefficients of the UPD for Two-Bay Truss in Configuration II**

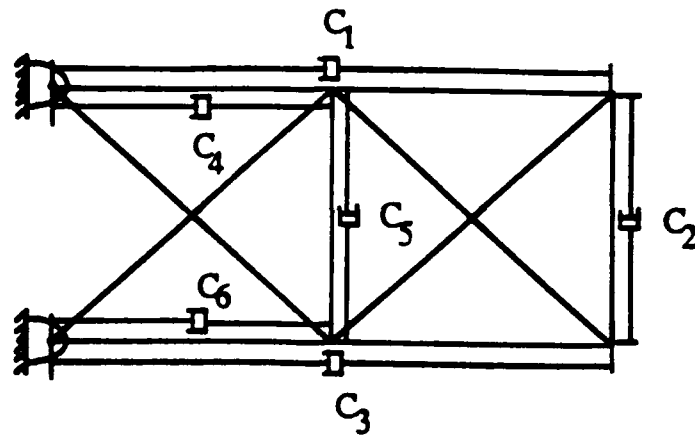
Damping Coeff. for Configuration II					
Matrix Q	$10 \cdot I$	$10^2 \cdot I$	$10^3 \cdot I$	$10^4 \cdot I$	$10^5 \cdot I$
$c_1$	1.5812	5.0001	15.8116	50.0006	158.1160
$c_2$	2.1092	6.6697	21.0915	66.6972	210.9151
$c_3$	1.5812	5.0001	15.8116	50.0006	158.1160
$c_4$	1.5815	5.0012	15.8151	50.0117	158.1510
$c_5$	1.0546	3.3348	10.5457	33.3485	105.4572
$c_6$	1.5815	5.0012	15.8151	50.0117	158.1510

**Table 3.3 Eigenvalues of Modified Two-Bay Truss with Damping Coefficients of Table 3.2**

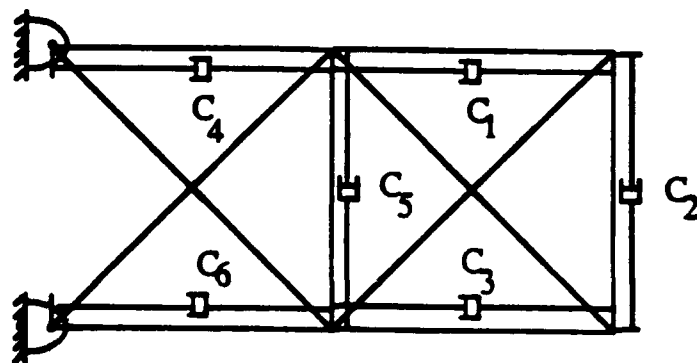
Eigenvalues of Modified Two-Bay Truss			
Matrix Q	$10^3 \cdot I$	$10^4 \cdot I$	$10^5 \cdot I$
Eigenvalue pairs	-0.28+143.97i	-0.88+143.97i	-2.78+143.93i
	-0.28+124.84i	-0.88+124.84i	-2.78+124.82i
	-0.23+123.06i	-0.72+123.06i	-2.26+123.04i
	-0.10+104.11i	-0.32+104.11i	-1.02+104.11i
	-0.03+87.27i	-0.09+87.27i	-0.29+87.27i
	-0.01+46.10i	-0.03+46.10i	-0.10+46.10i
	-0.02+42.69i	-0.08+42.69i	-0.25+42.69i
	-0.002+14.10i	-0.01+14.10i	-0.02+14.10i

**Table 3.4 Eigenvalues of Closed-Loop Two-Bay Truss with Different Controllers**

Eigenvalue Pairs				
LQR	ISDC	UPD/ISDC ( $Q=10^3 \cdot I$ )	UPD/ISDC ( $Q=10^4 \cdot I$ )	UPD/ISDC ( $Q=10^5 \cdot I$ )
-1.55+143.96i	-1.73+144.01i	-1.79+143.99i	-2.06+143.97i	-3.47+143.91i
-1.48+124.73i	-1.43+124.06i	-1.35+124.29i	-1.61+124.52i	-3.10+124.71i
-1.29+123.15i	-1.10+123.79i	-1.38+123.57i	-1.68+123.35i	-2.87+123.14i
-1.02+104.11i	-1.06+104.17i	-1.13+104.17i	-1.31+104.15i	-1.94+104.13i
-1.39+ 87.26i	-1.02+ 87.28i	-1.05+ 87.28i	-1.12+ 87.28i	-1.31+ 87.28i
-1.25+ 46.07i	-2.19+ 46.24i	-2.14+ 46.19i	-2.06+ 46.19i	-1.95+ 46.15i
-0.88+ 42.68i	-1.23+ 42.67i	-1.27+ 42.67i	-1.36+ 42.69i	-1.50+ 42.69i
-0.99+ 14.06i	-1.33+ 14.12i	-1.34+ 14.11i	-1.35+ 14.10i	-1.37+ 14.09i



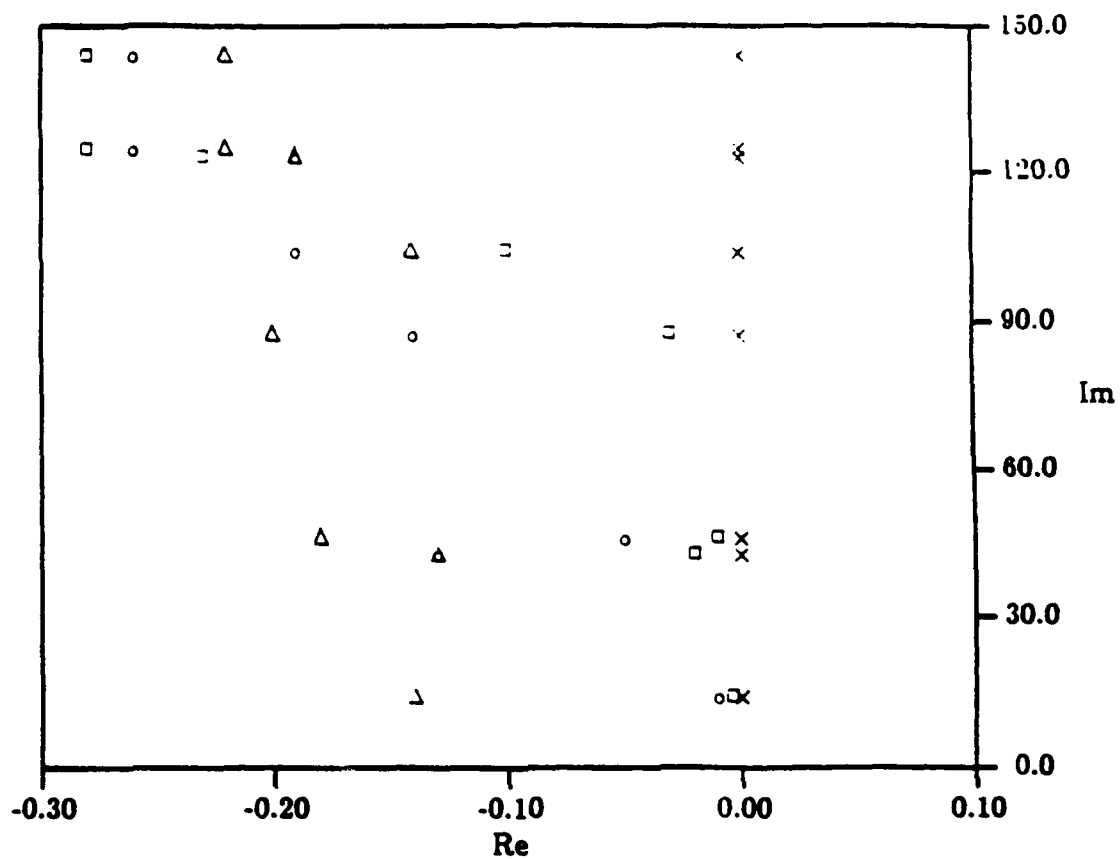
(a)



(b)

Figure 3.1 Damping Arrangement for Two-Bay Truss (a) Configuration I, (b) Configuration II

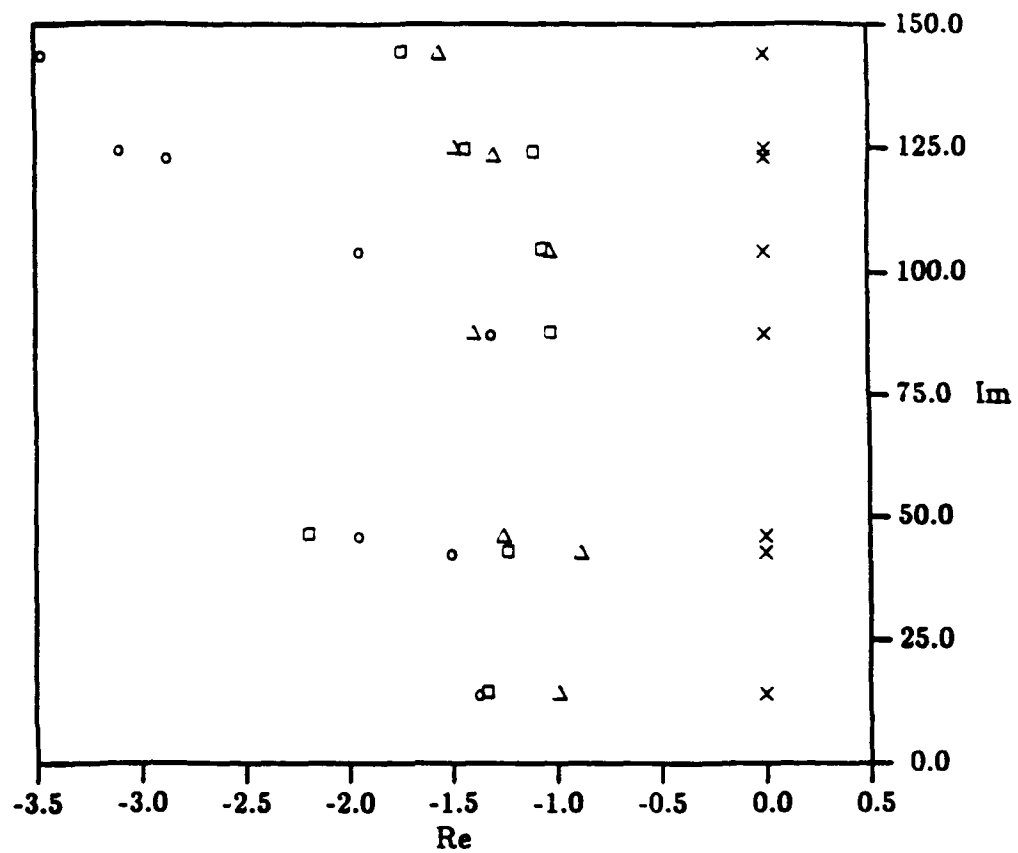




Legend :

- × : Eigenvalues of original structure
- Δ : Eigenvalues of closed-loop system with controller designed by LQR
- : Eigenvalues of modified system I (Fig. 3(a))
- : Eigenvalues of modified system II (Fig. 3(b))

Figure 3.2 Eigenvalues of Closed-Loop Controlled and Modified 2-bay Truss



Legend :

- × : Eigenvalues of open-loop structure
- Δ : Eigenvalues of closed-loop system with controller designed with complete model
- : Eigenvalues of closed-loop system with the ISDC controller
- : Eigenvalues of closed-loop system with multi-level UPD/ISDC controller

Figure 3.3 Eigenvalues of 2-bay Truss

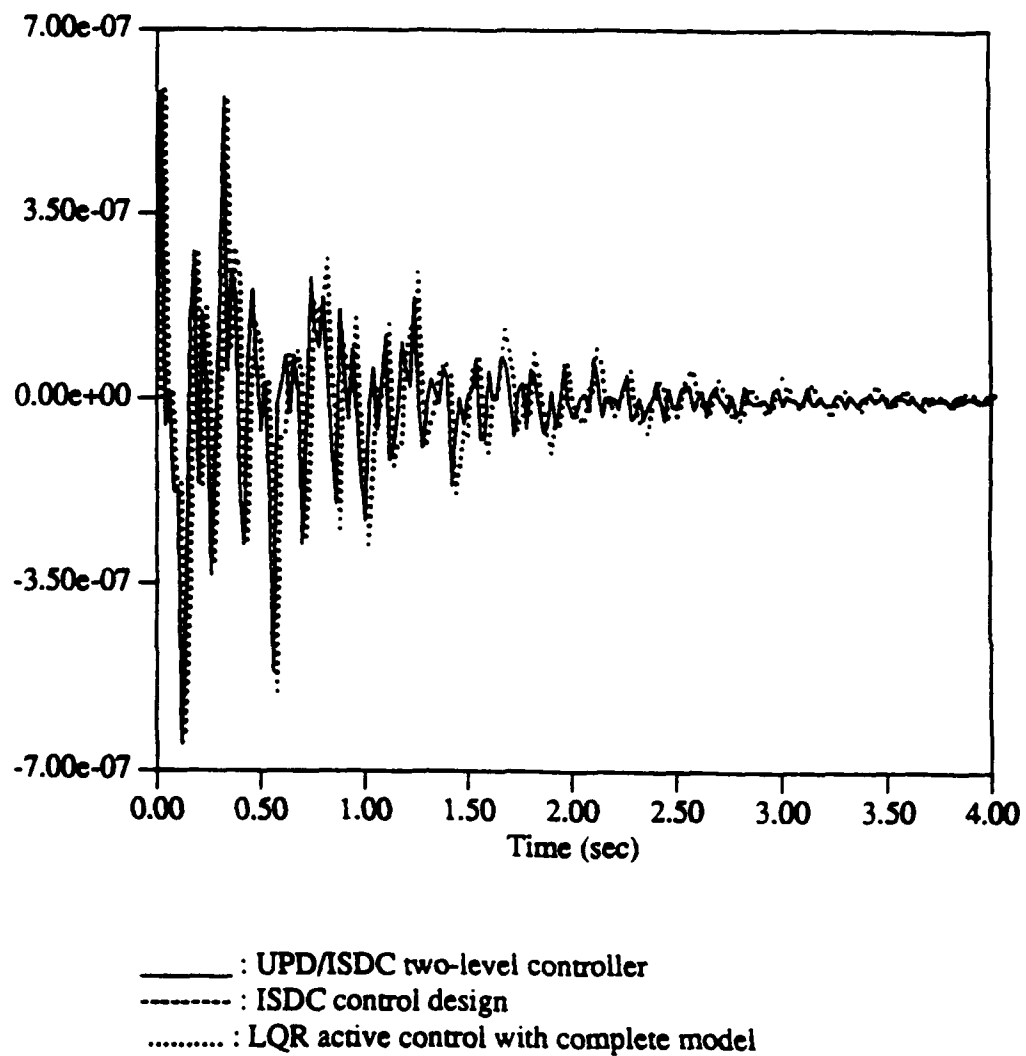
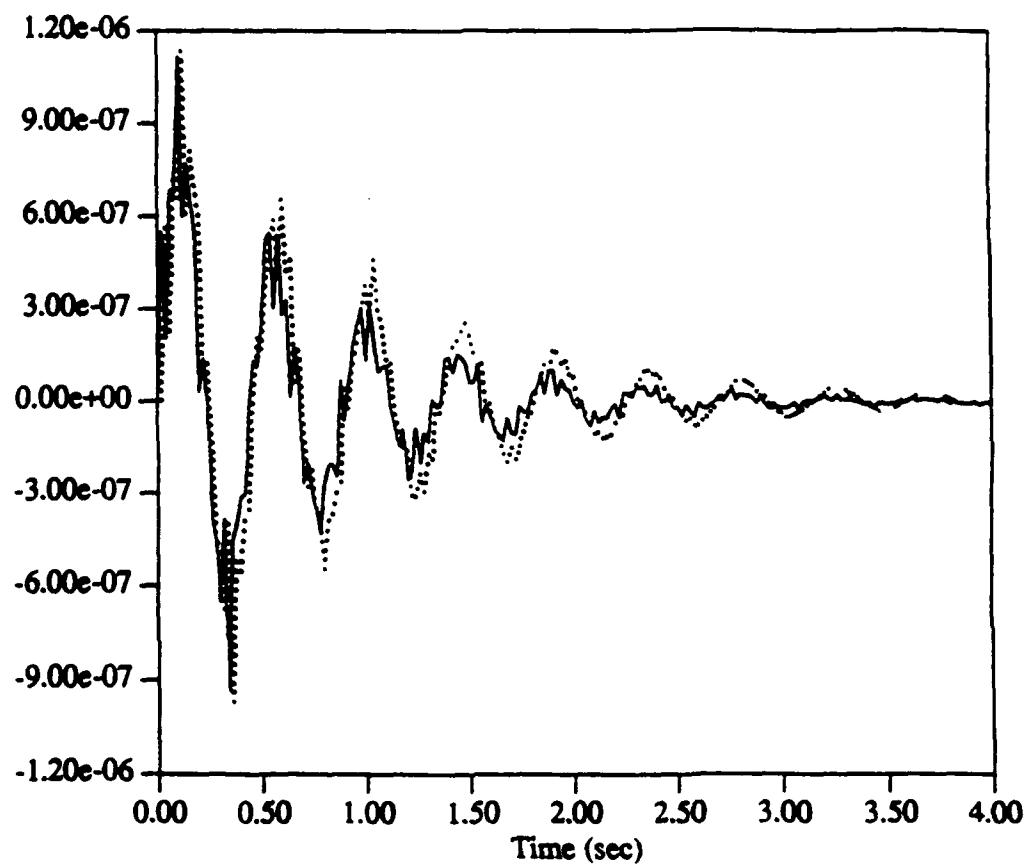


Figure 3.4 Displacement of 2-bay Truss at d.o.f. 1 (impulse at tip)



—— : UPD/ISDC two-level controller  
 - - - : ISDC control design  
 ..... : LQR active control with complete model

**Figure 3.5** Displacement of 2-bay Truss at d.o.f. 2 (impulse at tip)

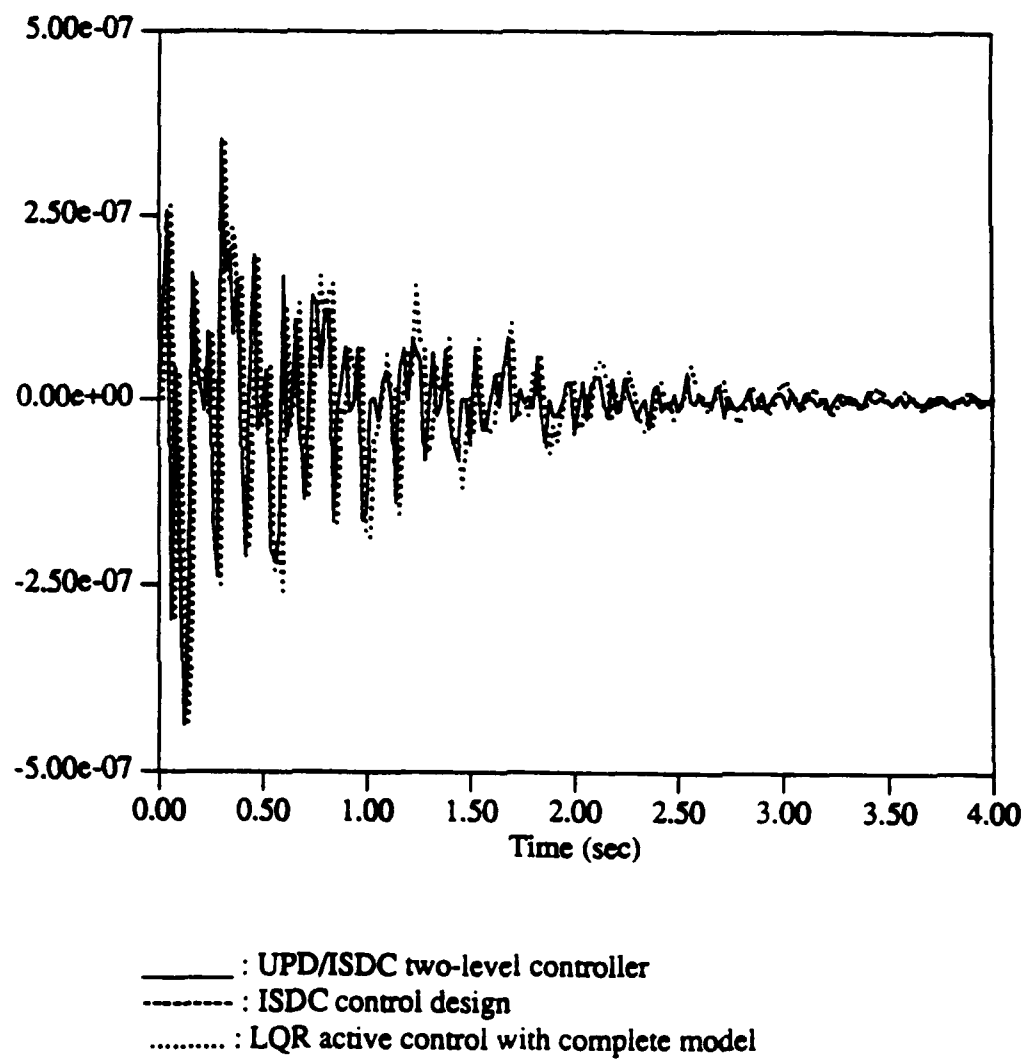
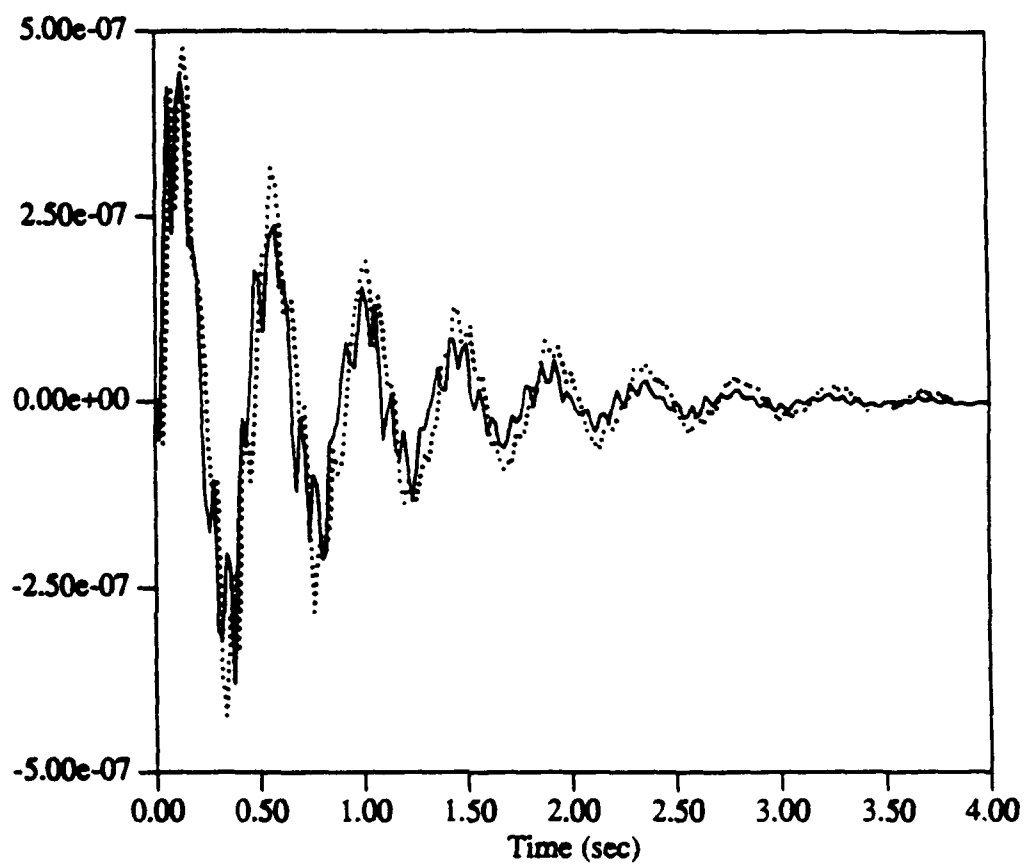
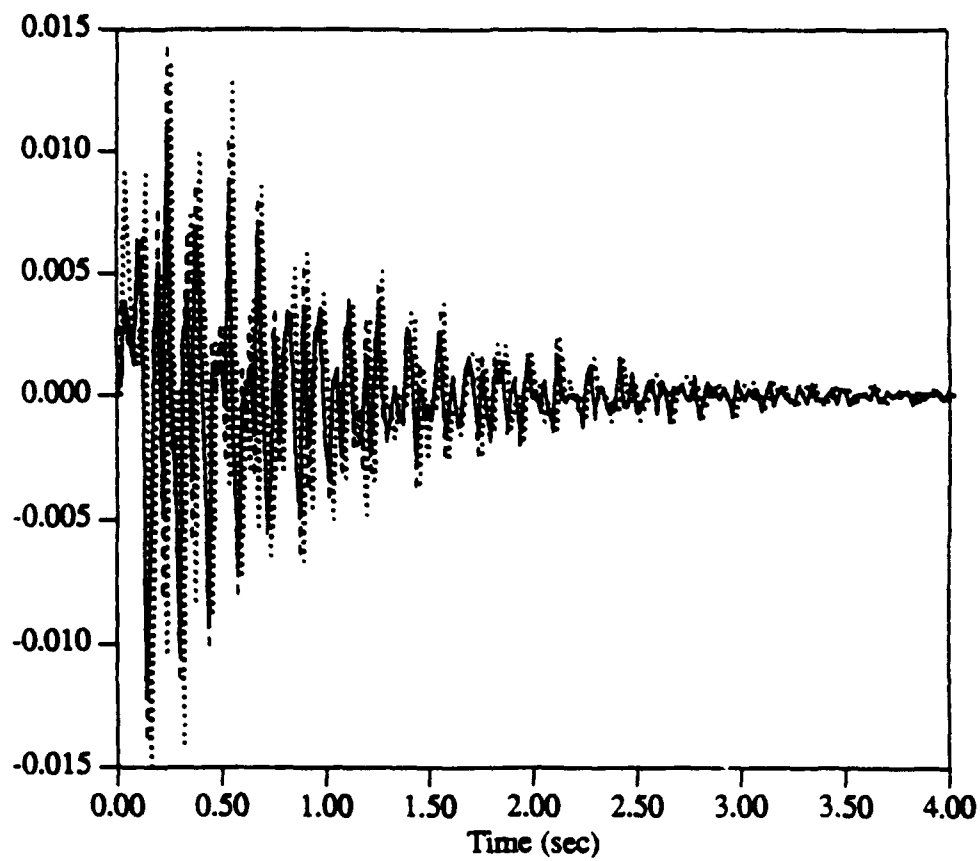


Figure 3.6 Displacement of 2-bay Truss at d.o.f. 5 (impulse at tip)



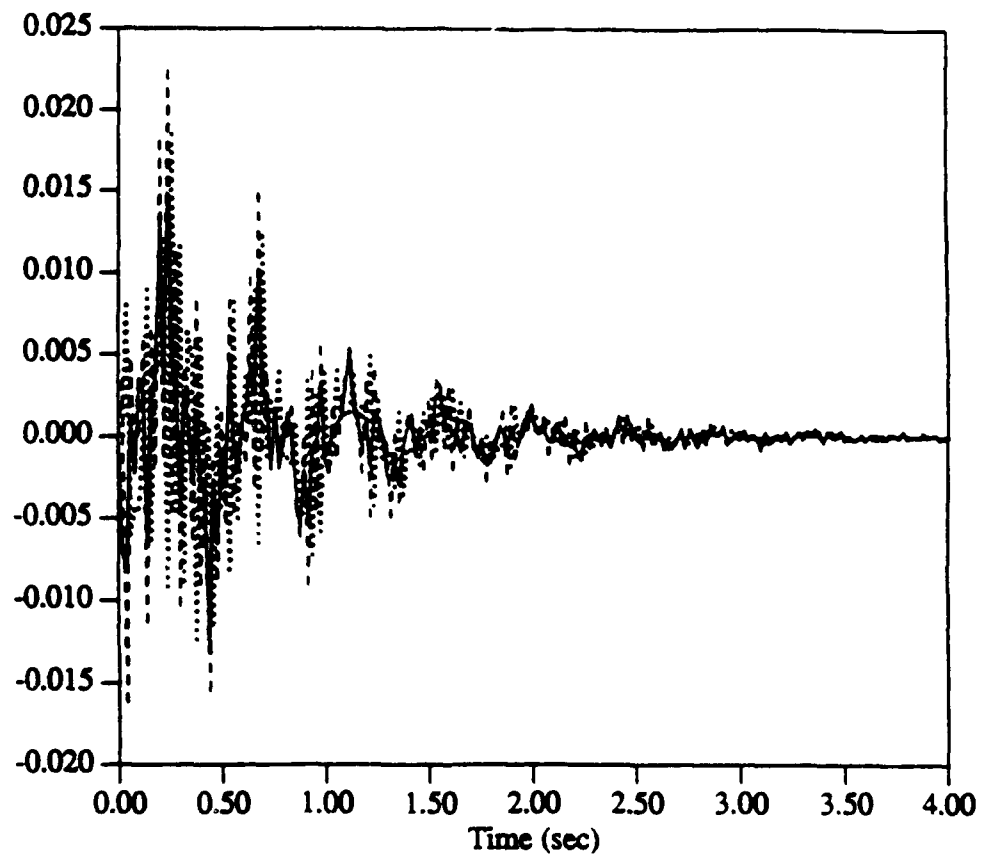
—— : UPD/ISDC two-level controller  
 - - - : ISDC control design  
 ..... : LQR active control with complete model

Figure 3.7 Displacement of 2-bay Truss at d.o.f. 6 (impulse at tip)



\_\_\_\_\_ : UPD/ISDC two-level controller  
 - - - - - : ISDC control design  
 ..... : LQR active control with complete model

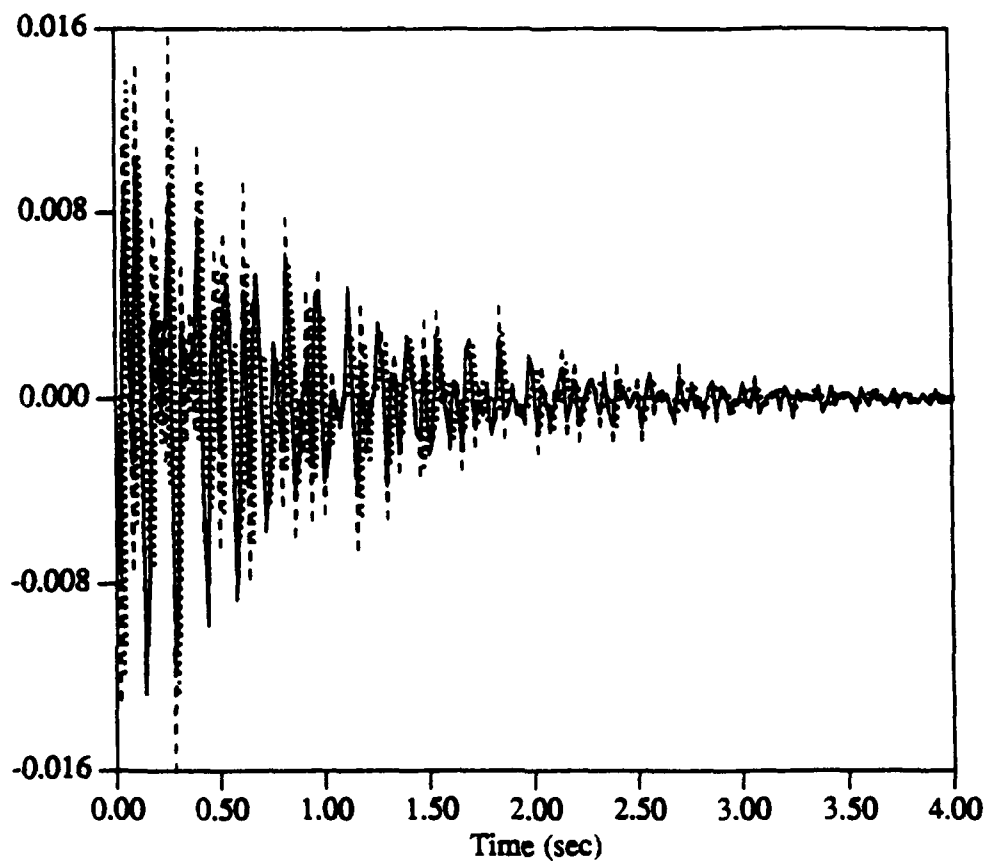
**Figure 3.8** Input Force of Actuator 1 of 2-bay Truss (impulse at tip)



— : UPD/ISDC two-level controller  
 - - - : ISDC control design  
 ..... : LQR active control with complete model

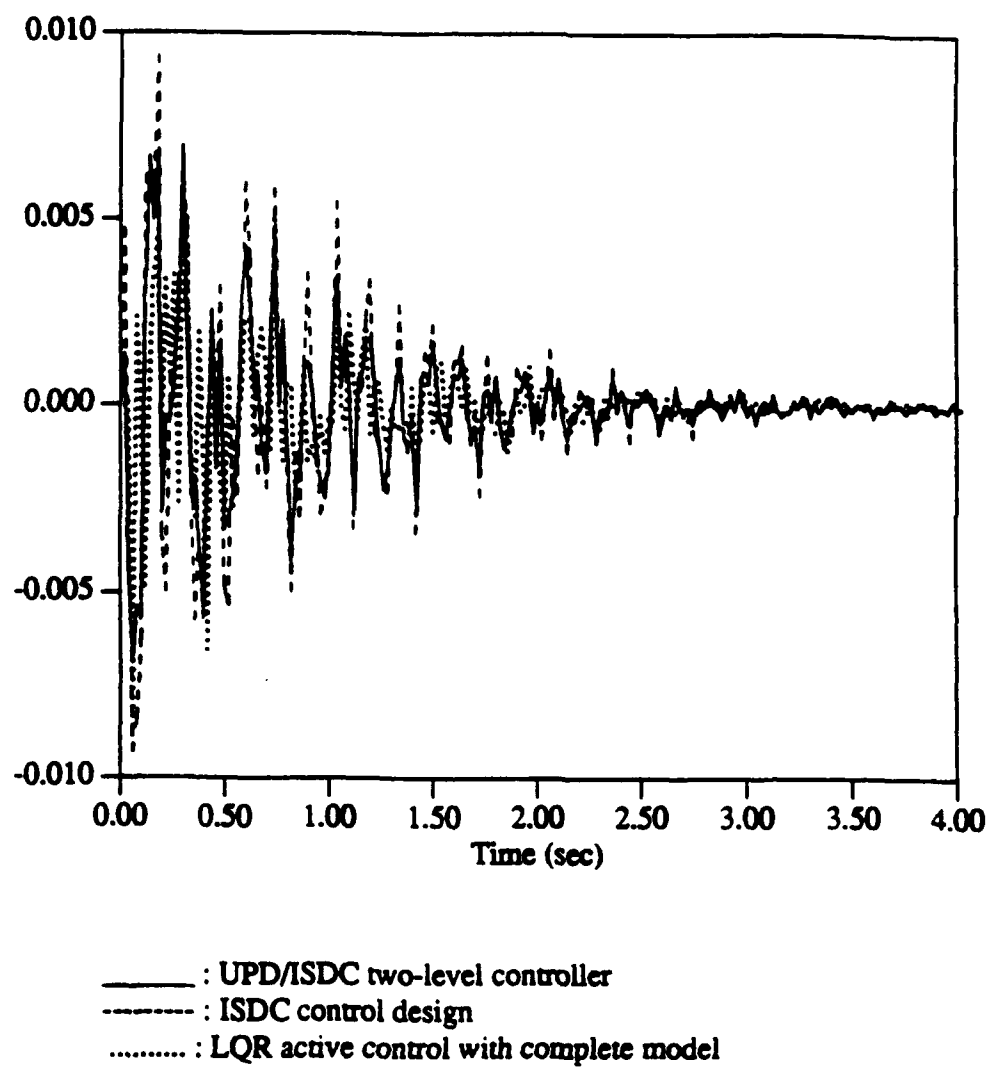
**Figure 3.9** Input Force of Actuator 2 of 2-bay Truss (impulse at tip)



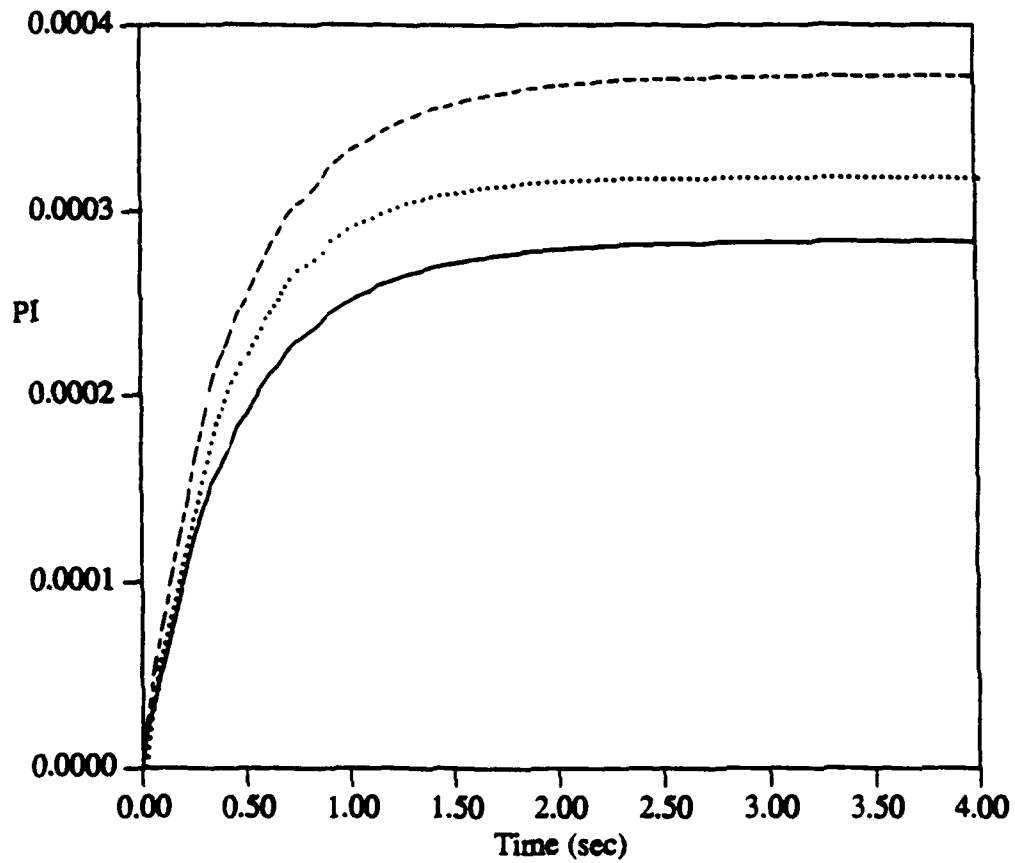


— : UPD/ISDC two-level controller  
 - - - : ISDC control design  
 ..... : LQR active control with complete model

**Figure 3.10** Input Force of Actuator 3 of 2-bay Truss (impulse at tip)



**Figure 3.11** Input Force of Actuator 4 of 2-bay Truss (impulse at tip)



—— : UPD/ISDC two-level controller  
----- : ISDC control design  
..... : LQR active control with complete model

Figure 3.12 Performance Index of 2-bay Truss (impulse at tip)

## CHAPTER 4

### ROBUST INTEGRATED STRUCTURAL AND CONTROL DESIGN

#### 4.1 Background

There has been a dramatic increase in the past decade in using active control systems to improve structural performance. The major challenge in the field of active control of structures is in the design of control systems for very large space structures. These structures are by nature distributed parameter systems with multiple inputs (controls) and a continuum of outputs (displacements). The finite element method is commonly used for the description of these structures. This is a source of parameter errors and truncated (or reduced order) models in the system. In addition, the structural properties of large space structures cannot be tested before they are put into orbit and hence sizeable uncertainties exist in modal parameters.

A great deal of research is currently in progress on developing methods for the simultaneous (integrated) design of the structure and the control system. The weight of the structure was minimized with constraints on the distribution of the eigenvalues and/or damping ratio of the closed-loop system by Khot et al. (1985). Miller and Shim (1986) considered the simultaneous minimization, in structural and control variables, of the sum of structural weight and the infinite horizon linear regulator quadratic control cost. The structure/control system optimization problem was formulated by Khot, et. al (1986) with constraints on the closed-loop eigenvalue distribution and the minimum Frobenious norm of the control gains. In all the above works, the consideration of robustness of the control system has been ignored.

The parameter variations introduced by the analysis model, uncertain material properties or optimization may adversely influence the stability and performance characteristics of the control system. The robustness is an extremely important feature of a feedback control design. A robust control design is one which satisfactorily meets the system specifications even in the presence of parameter uncertainties and other modeling errors. Since the system specifications could be in terms of stability and/or performance, two types of robustness, namely, stability robustness and performance robustness, are to be considered in the design stage.

The current published literature on control system robustness addresses either the stability robustness aspect or the performance robustness aspect. Most of the work on the stability robustness (in the controls area) was done in the frequency domain using singular value decomposition while much of the useful research on performance robustness was carried out in time domain using sensitivity approaches. Design studies that treated the stability robustness aspect in time domain and studies which combined both stability robustness and performance robustness in the design process have been scarce. The recent developments in the area of robust multivariable control theory have been summarized by Ridgely and Banda (1986). The

stability robustness of linear systems was analyzed in the time domain in Yedavalli et al.(1985) wherein a bound on the perturbation of an asymptotically stable linear system was obtained to maintain stability using Lyapunov matrix equation solution. In Kosut et al.(1983), singular value robustness measures were used to compare the performance and stability robustness properties of different control design techniques in the presence of residual modal interaction for a flexible spacecraft system. The importance of robustness considerations in the design of flexible space structures was discussed by Hoehne (1985). Gordon and Collins (1985) presented a direct design method for solving the problem of robustness to cross-coupling perturbations by treating the feedback gains as design variables. Their method makes use of nonlinear programming techniques along with a time domain pole placement procedure. A technique for the improvement of stability robustness by shaping the singular value spectrum using constrained optimization methods was described in Mukhopadhyay (1985).

## 4.2 Robustness Analysis in Controlled Structures

Stability robustness and performance robustness are two very important properties which give the ability of the closed-loop system to maintain its asymptotic stability and performance requirements under perturbations or errors in the system model or controller parameters. Hence, the measures of the stability and performance robustness are important. The stability robustness index, a measure of closed-loop system stability under uncertainty or modeling errors, is defined as the summation of weighted variations of system eigenvalues and can be written as

$$\beta_{sr} = \sum_{i=1}^{2n} w_i |\lambda_i - \bar{\lambda}_i| \quad (4.1)$$

where  $\beta_{sr}$  is the stability robustness index, and  $\lambda_i$  and  $\bar{\lambda}_i$  are the closed-loop eigenvalues of nominal system and perturbed system, respectively. The weighting coefficient  $w_i$ , denoting the importance of eigenvalue pair  $i$ , is determined by the horizontal distance of eigenvalue to imaginary axis in complex plane:

$$w_i = \frac{1}{S * \text{real}(\lambda_i)} \quad (4.2)$$

in which

$$S = \sum_{i=1}^{2n} \frac{1}{\text{real}(\lambda_i)} \quad (4.3)$$

In addition to stability robustness, it is desirable to retain the performance unchanged when the design variables are changed. Since the performance cannot remain the same, a performance

robustness index is defined to present the performance robustness. From optimal linear quadratic regulator, if  $P$  is the steady-state solution of matrix Riccati equation, the performance index is given as a function of initial state vector as

$$J = x_0^T P x_0 \quad (4.4)$$

The performance index for perturbed system with the same controller can be written as

$$\bar{J} = x_0^T \bar{P} x_0 \quad (4.5)$$

where  $\bar{P}$  satisfies the Lyapunov equation

$$\bar{A}_{cl}^T \bar{P} + \bar{P} \bar{A}_{cl} + G^T R G + Q = 0 \quad (4.6)$$

in which,  $\bar{A}_{cl}$  is perturbed closed-loop system matrix:

$$\bar{A}_{cl} = \bar{A} + \bar{B} G \quad (4.7)$$

$R$  and  $Q$  are positive definite input and positive semi-definite output matrices and  $G$  is the gain matrix for controller:

$$G = -R^{-1} B^T P \quad (4.8)$$

and  $P$  satisfies the relation

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \quad (4.9)$$

The performance robustness index is defined as the error ratio of steady-state performance index:

$$\beta_{pr} = \left| \frac{J - \bar{J}}{J} \right| = \frac{x_0^T (P - \bar{P}) x_0}{x_0^T P x_0} \quad (4.10)$$

It has been pointed out by Kleinman and Athans (1968) that the optimal solution of Eq.(4.4) in general depends on the initial state  $x_0$ . This result is not very useful since the initial state is not always known. The effect of  $x_0$  is averaged out by assuming that  $x_0$  is a uniformly distributed random vector whose covariance is given by the identity matrix. The trace of  $P$  is proportional to the expected value of  $J$ . Hence Eq.(4.10) can be written as

$$\beta_{pr} = \left| \frac{\text{Tr}(P) - \text{Tr}(\bar{P})}{\text{Tr}(P)} \right| \quad (4.11)$$

For both nominal and perturbed systems, good dynamic response can be achieved if the real part of every eigenvalue is restricted to be smaller than a specified value as  $\text{Re}(\lambda) < -a$ , with  $a > 0$ . The LQ regulator can be modified (Jacobson et al., 1980) as follows with the requirement of (A,B) being controllable and (A,C) being observable.

$$\dot{x} = (A + aI)x + Bu = (A + aI - BG)x \quad (4.12)$$

$$u = -R^{-1}B^TK_a x = -Gx \quad (4.13)$$

with  $K_a$  satisfying the equation

$$(A + aI)^TK_a + K_a(A + aI) - K_aBR^{-1}B^TK_a + C^TQC = 0 \quad (4.14)$$

The characteristic equation of Eq.(4.12) is given by

$$\det [(A + aI - BG) - \lambda I] = \det [A - BG - \lambda' I] = 0 \quad (4.15)$$

with  $\lambda' = \lambda - a$ . Since A is assumed to be stable, then  $\text{Re}(\lambda') < 0$ , and hence  $\text{Re}(\lambda) < -a$ . Thus, the system with controller, Eq.(4.13), provides a guaranteed stability margin.

### 4.3 Multiobjective Design Problem

The stability robustness, the performance robustness and the total structural weight are considered as the objective functions in this work. The cross sectional areas of the members are treated as the design variables. The first objective function, the stability robustness index ( $\beta_{sr}$ ), describes the relative stability of the system when the design variables change by a specified amount. It is assumed that the controller gains are such that the condition for the stability of the system is satisfied and thus the closed-loop system matrix of the perturbed system is still stable. According to this definition,  $\beta_{sr}=0$  corresponds to a highly robust system from the stability point of view. However,  $\beta_{sr}$  will not attain the value zero because of the presence of perturbations in the design variables. The second objective function, the performance robustness index of the system ( $\beta_{pr}$ ), is defined by Eq.(4.11). Here also,  $\beta_{pr}=0$  corresponds to a highly robust system from the performance point of view. The value  $\beta_{pr}=0$  will not be attained in practice because of the presence of perturbations in the design variables. The third design objective function, the total structural weight, is given by

$$f_3(x) = \sum_{i=1}^N \rho_i l_i A_i \quad (4.16)$$

with  $\rho_i$ ,  $l_i$  and  $A_i$  denoting the density, length and cross-sectional area of the  $i^{\text{th}}$  member respectively, and N representing the number of members in the truss structure. The following constraints are used during the optimization procedure:

1. Upper and lower bounds on the design variables.
2. Stability requirement, i.e., the requirement of the real parts of the eigenvalues of the closed-loop system to be negative.
3. Lower and/or upper bounds on the natural frequencies of vibration of the structure.

In some cases, the closed-loop damping ratios of the system may have to be constrained; however, these are not considered in this work.

#### 4.4 Multiobjective Optimization Techniques

The three objective optimization problem formulated is solved using the utility function, the lexicographic and the goal programming methods. The utility function method involves the solution of the following problem (Rao, 1984A):

$$\text{Min } U(x) = \sum_{i=1}^k w_i f_i(x) \quad (4.17)$$

subject to

$$g_j(x) \leq 0, \quad j=1,2,\dots,m$$

where  $w_i$  is the weight of the  $i^{\text{th}}$  objective function  $f_i$  and  $\sum_{i=1}^k w_i = 1$ . Usually, the scales and units of different objective functions are different. Hence a suitable normalization process has to be used in constructing the objective functions of Eq.(4.17). A convenient form is to define a new objective function  $F_i$  as

$$F_i(x) = \frac{f_i(x) - f_i^*(x_i^*)}{f_i^*(x_i^*)} \quad (4.18)$$

where  $x_i^*$  is the optimal solution for individual objective function  $f_i$ , and  $f_i^*(x_i^*)$  is optimal function value of objective function  $i$ . Redefine the objective function of Eq.(4.17) as

$$\text{Min } U(x) = \sum_{i=1}^k w_i F_i(x) \quad (4.19)$$

In the lexicographic method, the objectives are ranked in order of importance by the designer. The preferred solution obtained by this method is one which minimizes the objectives starting with the most important one and proceeding according to the order of importance of the objectives. Let the subscripts of the objectives indicate not only the objective function number, but also the priorities of the objectives. Thus  $F_1(x)$  and  $F_k(x)$  denote the most and the least important objective functions, respectively. Then the first problem is formulated as

$$\text{Min } F_1(x) \quad (4.20)$$

subject to



$$g_j(x) \leq 0, j=1,2,\dots,m$$

and its solution  $x_1^*$  and  $F_1^* = F_1(x_1^*)$  are obtained. Then the second problem is formulated as

$$\text{Min } F_2(x) \quad (4.21)$$

subject to

$$g_j(x) \leq 0, j=1,2,\dots,m, \text{ and}$$

$$g_{m+1}(x) = F_1(x) - F_1(x_1^*) \leq \epsilon_1$$

where  $\epsilon_1$  is a small value compared to  $F_1(x_1^*)$ . This procedure is repeated until all  $k$  objectives have been considered. The  $i^{\text{th}}$  problem is given by

$$\text{Min } F_i(x) \quad (4.22)$$

subject to

$$g_j(x) \leq 0, j=1,2,\dots,m, \text{ and}$$

$$g_{m+n}(x) = F_n(x) - F_n(x_n^*) \leq \epsilon_n, n=1,2,\dots,i-1$$

The solution obtained at the end, i.e.  $x_k^*$  is taken as the desired solution  $x^*$  of the multiobjective optimization problem.

The goal programming method was originally proposed by Charnes and Cooper (1977) for linear optimization problems. The method requires goals to be set for each objective that the designer wishes to obtain. A preferred solution is then defined as the one which minimizes the deviations from the set goals. Thus a simple goal programming problem can be defined as

$$\text{Min } F(x) = \left[ \sum_{i=1}^k (F_i(x))^p \right]^{1/p}, p \geq 1 \quad (4.23)$$

subject to

$$g_j(x) \leq 0, j=1,2,\dots,m$$

$$\tilde{F}_j(x) \geq 0, j=1,2,\dots,k$$

where  $\tilde{F}_j(x) = F_j(x) - F_j(x_j^*)$ .

## 4.5 Computational Procedure

### 4.5.1 Analysis

The following analysis procedure is used to study the effects of variations in the parameters of the structure on the robustness of the system:

1. Start with an initial reference design of the structure and find the corresponding plant matrix A, input matrix B and the output matrix C.
2. Use the LQ regulator design technique to find the optimal control gain G by solving the algebraic Riccati equation.
3. Change the design parameters by known percentage values and find the corresponding A, B, and C matrices.
4. Find the stability robustness index  $\beta_{sr}$  (Eq.(4.1)) and the performance robustness index  $\beta_{pr}$  (Eq.(4.11)).
5. Repeat steps (3) and (4) for different parameter changes. (e.g., nominal design variables, damping ratio, density etc.)
6. Plot a graph between  $\beta_{sr}$  or  $\beta_{pr}$  and the change in the parameters.

#### 4.5.2 Design

The purpose of design is to optimize the actively controlled structure by using suitable multiobjective optimization techniques. The procedure is given as follows:

1. From the requirements of stress and deformation, obtain the preliminary design (to be used as the nominal design) of the structure.
2. Construct the plant matrix, input matrix, and output matrix.
3. Formulate the multiobjective constrained optimization problem.
4. Minimize the individual objective functions and find the respective minima around the nominal design.
5. Use a suitable multiobjective optimization approach to find a compromise solution.

#### 4.6 Examples

##### 4.6.1 Two-Bar Truss

The two-bar truss shown in Fig. 4.1 is selected for its simplicity. A nonstructural mass of 1 unit is attached at node 3. The actuators and the sensors are colocated at node 3 acting in x and y directions. The design variables (cross-sectional areas of the two bars) are restricted to lie between 0.01 and 1.0. The structural damping ratio is considered as 0.01, Young's modulus is assumed to be  $10^7$ , and density is taken to be 4.6. In the performance index, the output weighting matrix Q is assumed to be  $1000 \cdot I$ , and the input weighting matrix R is taken to be I, where I is the identity matrix. The natural frequencies of the closed-loop controlled structure are constrained to lie between 20 rad/sec and 40 rad/sec. For a stable open loop system, the corresponding feedback closed-loop system must be stable under the optimum control law. But the stability is not guaranteed if there exist disturbances or uncertainties in system parameters. Hence, additional constraints are added on the perturbed closed-loop system, namely, that all the eigenvalues of the perturbed closed-loop system are restricted to have negative real parts.

### Analysis:

Figure 4.2 shows the relationship between the stability robustness index and the design variables, which is a smooth concave function. Figure 4.3 shows the variation of the performance robustness index with the two design variables; it is a non-concave function having several local minima in the design space. Figure 4.4 shows the variations of stability robustness index and performance robustness index with changes in the structural damping ratio of the two-bar truss. The stability robustness index drops sharply at a value of the damping ratio of approximately 4%. The performance robustness index can be observed to attain a minimum value at a damping ratio of approximately 1%. Figure 4.5 shows that the system performance index ( $J$ ) decreases as the damping ratio increases. The effect of the variations in Young's modulus of the material on the robustness indices is shown in Figure 4.6. The stability robustness index monotonically increases as the Young's Modulus increasing. The performance robustness index reduces to a minimum value at Young's modulus ( $E$ )= $20 \times 10^6$  and then increases for larger values of  $E$ . Figure 4.7 indicates that  $J$  increases to a maximum value at Young's modulus  $E=20 \times 10^6$  then decreases for larger values of  $E$ .

The relationship of  $\beta_{sr}$  and  $\beta_{pr}$  with the mass density of the material is shown in Fig. 4.8. The stability robustness index decreases with an increase in the density of the material. The performance robustness index reduces to a minimum at  $\rho=1.5$  and then increases for higher values of  $\rho$ . In Fig. 4.9, the performance index attains a maximum value at  $\rho=1.5$  and then decrease monotonically as the material density increases. Figure 4.10 shows the variations of the stability robustness index and the performance robustness index with a change in the coefficient of the output weighting matrix. An increase in the coefficient of the output weighting matrix implies that the output performance is more important than the control energy. The minimum of  $\beta_{sr}$  and the maximum of  $\beta_{pr}$  occur at coefficient=400, and a larger coefficient increases the system stability robustness index but reduces the system performance robustness index when the coefficient is greater than 400. Figure 4.11 shows that the performance index reaches the minimum when the coefficient of output weighting matrix is equal to 400 and monotonically increases when the coefficient is greater than 400.

### Design:

The results of minimization of the individual objective functions are shown in Table 4.1. The results given by different multiobjective optimization methods are shown in Table 4.2. The first two columns in Table 4.2 correspond to formulations #1 and #2 of the utility function method. In formulation #1,  $w_i$  are set equal to a fixed value of  $1/3$  in Eq.(2.17) while  $w_i$  are considered as design variables in formulation #2. The last row of Table 4.2 gives the values of the global evaluation function,  $F_g$ , which can be used as an index to compare the results of different multiobjective optimization methods. The global evaluation function is defined as

$$F_g(x) = \sum_{i=1}^3 F_i(x^*) \quad (4.24)$$

where

$$F_1(x) = \frac{(f_1(x) - 0.009502)}{0.010535},$$

$$F_2(x) = \frac{(f_2(x) - 0.0015899)}{0.0032231},$$

and  $F_3(x) = \frac{(f_3(x) - 23.598)}{43.682}.$

#### 4.6.2 Two-Bay Truss

The finite element model of the second example (two-bay truss) is shown in Fig. 4.12. For this example, nonstructural masses of magnitude 1.29 are attached at nodes 1 to 4. Each node has two degrees of freedom. The actuators and sensors are colocated at nodes 1 to 4 and are assumed to act along the y-direction only. The design variables (cross-sectional areas) are restricted to lie between 0.001 and 0.5. The natural frequencies of the closed-loop system are constrained to be larger than 31.62 rad/sec (i.e.,  $\omega^2 \geq 1000$ ).

Analysis:

This example has 10 design variables. Since the display of functional relations in 10-dimensional design space is not possible, the variation of the robustness of the system is found by uniformly varying the value of all the 10 design variables. The results are shown in Fig. 4.13. This figure shows the stability robustness index versus the value of the design variables when the permissible change in the design vector is assumed to be -5%.  $\beta_{sr}$  and  $\beta_{pr}$  decrease slowly with an increase in the value of the design variables. The  $\beta_{sr}$  shows a smooth concave function, while the  $\beta_{pr}$  shows a nonsmooth curve. Hence local minima are expected in the optimization process for the performance robustness index. In Fig. 4.14, the performance index decreases monotonically with increase in the value of the design vector. This implies that a stronger structure will induce a smaller displacement and needs lesser control energy to obtain good performance.

Design:

The nominal values of the design variables are  $x_i = 0.1$ ,  $i=1$  to 10. Table 4.3 gives the results obtained by optimizing the individual objective functions starting from the nominal design. The results of different multiobjective optimization methods, namely, the utility function method, the lexicographic method and the goal programming method are compared in

Table 4.4. The last row of Table 4.4 shows the global evaluation function,  $F_g$ , defined as

$$F_g(x) = \sum_{i=1}^3 F_i(x^*) \quad (4.25)$$

where

$$F_1(x) = \frac{(f_1(x) - 0.048694)}{0.000668},$$

$$F_2(x) = \frac{(f_2(x) - 0.008454)}{0.001588},$$

and  $F_3(x) = \frac{(f_3(x) - 0.2273)}{0.02681}.$

#### 4.7 Concluding Remarks

The stability robustness index and the performance robustness index defined in this chapter are highly nonlinear with respect to design variables. The non-concave property of robustness indices with changes in design variables leads to difficulties in optimization. As such one can expect to find only a local optimum in the neighborhood of the starting design during optimization. In general, the local optima are acceptable since the starting design is usually taken as the nominal design which is expected to be robust. Techniques, such as genetic algorithms and simulated annealing, might be alternatives to avoid the local minima to obtain the global optimal design. The relationships between the stability/performance robustness index and the various system parameters have been determined numerically for the two-bar truss. These results are expected to be useful in choosing suitable material for a given structure with a specified geometry or weighting coefficients in the performance index for controller design.

A major advantage of using nonlinear programming to find the robust control/structural design is that it can be used with large permissible changes in the design variables and/or different constraint specifications. Three multiobjective optimization methods have been used to find the optimal designs of the illustrative examples. For the two-bar truss, the utility function method with variable coefficients gave the smallest value of the global evaluation function. For the two-bay truss, the goal programming method with  $p=2$  yielded the smallest value for the global evaluation function and the utility function method with variable coefficients gave the second smallest value. As observed in the investigation, no particular method gives the best solution for all the problems. Hence, several methods are to be used to solve the problem and find the best trade-off between the multiple objectives.

Table 4.1

## Single Objective Optimization of Two-Bar Truss.

Permissible design variable change = -5%.

$$\xi = 0.01, \quad \zeta = 10^3, \quad x(o) = \begin{Bmatrix} 0.1 \\ 0.1 \end{Bmatrix}$$

Minimization of			
Objective	$\beta_{sr}$	$\beta_{pr}$	Weight, W
$C_i$ $i=1,3$	1.0 0. 0.	0. 1.0 0.	0. 0. 1.0
$x^*$	0.14626 0.14626	0.15247 0.13797	0.051301 0.051301
$f_1(x^*)$	0.009502	0.009557	0.020037
$f_2(x^*)$	0.001618	0.0015899	0.004813
$f_3(x^*)$	67.28	66.801	23.598
$f^* = \sum_{i=1}^3 C_i f_i$	0.009502	0.0015899	23.598

Table 4.2

## Multiobjective Optimization of Two-Bar Truss

Permissible design variable change = -5%

$$\xi = 0.01, \quad \zeta = 10^3, \quad x(o) = \begin{Bmatrix} 0.1 \\ 0.1 \end{Bmatrix}$$

	Utility Function Method		Lexicographic Method			Goal Programming Method	
	Const. Coef.	Variable Coef.	Optimization Order			p=1	p=2
			$f_1, f_2, f_3$	$f_3, f_1, f_2$	$f_2, f_1, f_3$		
Optimal Design Variables $X^*$	$X_1 = 0.1309$	0.1463	0.15389	0.056296	0.14466	0.051294	0.051807
	$X_2 = 0.12709$	0.1299	0.13483	0.056303	0.1440	0.051309	0.051807
$f_1(X^*)$	0.010506	0.009950	0.009603	0.018932	0.009604	0.020037	0.019919
$f_2(X^*)$	0.001867	0.001792	0.001669	0.004399	0.001664	0.004809	0.004770
$f_3(X^*)$	59.337	63.549	66.404	25.898	66.391	23.599	23.831
$\sum_{i=1}^3 F_i(X^*)$	0.999438	0.977291	1.014075	1.819317	1.010398	1.998782	1.980792

Table 4.3

## Single Objective Optimization of Two-Bay Truss

Permissible design variable change = -5%

$$\xi = 0.01, \quad \zeta = 10^3, \quad X_i(o) = 0.1 \quad i=1 \text{ to } 10$$

Minimization of			
Objective	$\beta_{sr}$	$\beta_{pr}$	Weight, W
Optimal Design Variables $X_i^*$ $i=1$ to 10	0.13816	0.13765	0.11274
	0.09648	0.09339	0.00100
	0.13782	0.13777	0.11315
	0.27661	0.27637	0.33788
	0.10103	0.09899	0.00100
	0.27780	0.27725	0.33772
	0.14537	0.14671	0.11810
	0.14417	0.14507	0.11884
	0.14808	0.14998	0.12110
	0.15119	0.14920	0.12122
$f_1(X^*)$	0.048694	0.048701	0.049354
$f_2(X^*)$	0.010019	0.00981975	0.0084567
$f_3(X^*)$	0.252557	0.252307	0.22730



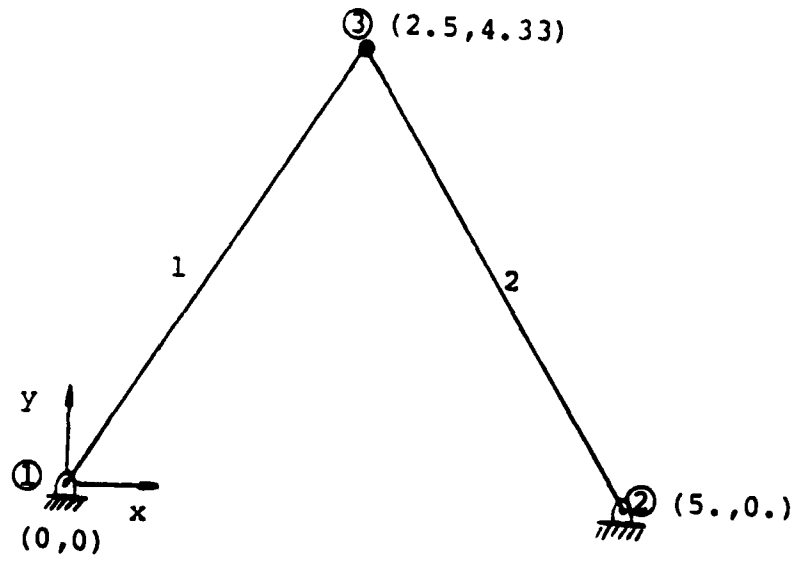
Table 4.4

## Multiobjective Optimization of Two-Bay Truss

Permissible design variable change = -5%

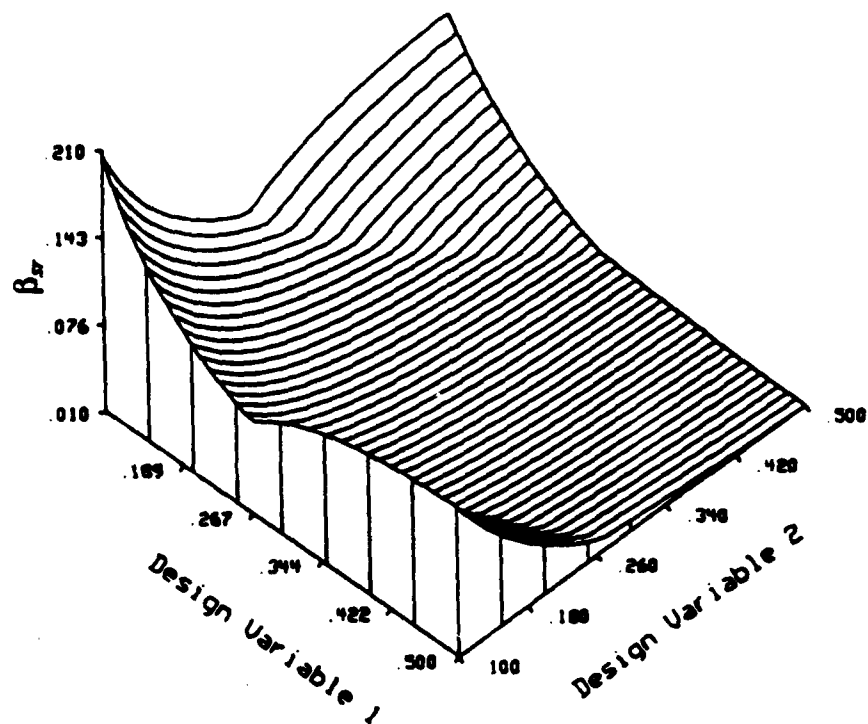
$$\xi = 0.01, \quad \zeta = 10^3, \quad X_i(o) = 0.1 \quad i = 1, 10$$

Approach	Utility Function Method		Lexicographic Method		Goal Programming Method
	Const. Coef.	Variable Coef.	Optimization Order		p=2
			$f_1, f_2, f_3$	$f_3, f_1, f_2$	
Optimal Design Variables $X_i^*$ $i=1, 10$	0.12247	0.13753	0.13697	0.14197	0.11389
	0.01779	0.09207	0.09588	0.03602	0.00100
	0.11406	0.13603	0.15071	0.16162	0.11142
	0.32120	0.27907	0.29918	0.36978	0.00100
	0.01000	0.09880	0.07856	0.00100	0.00100
	0.33931	0.27920	0.29422	0.34327	0.33617
	0.10835	0.14554	0.15018	0.11642	0.11907
	0.12838	0.14278	0.15061	0.12668	0.11901
	0.13045	0.14917	0.10706	0.10488	0.12277
	0.11834	0.14972	0.12839	0.11305	0.12420
$f_1(X^*)$	0.048769	0.048742	0.048788	0.048664	0.049360
$f_2(X^*)$	0.012034	0.00997	0.010307	0.011721	0.008479
$f_3(X^*)$	0.22903	0.25196	0.24841	0.24438	0.22736
$\sum_{i=1}^3 F_i(X^*)$	2.431212	1.946322	2.094988	2.694381	0.999241

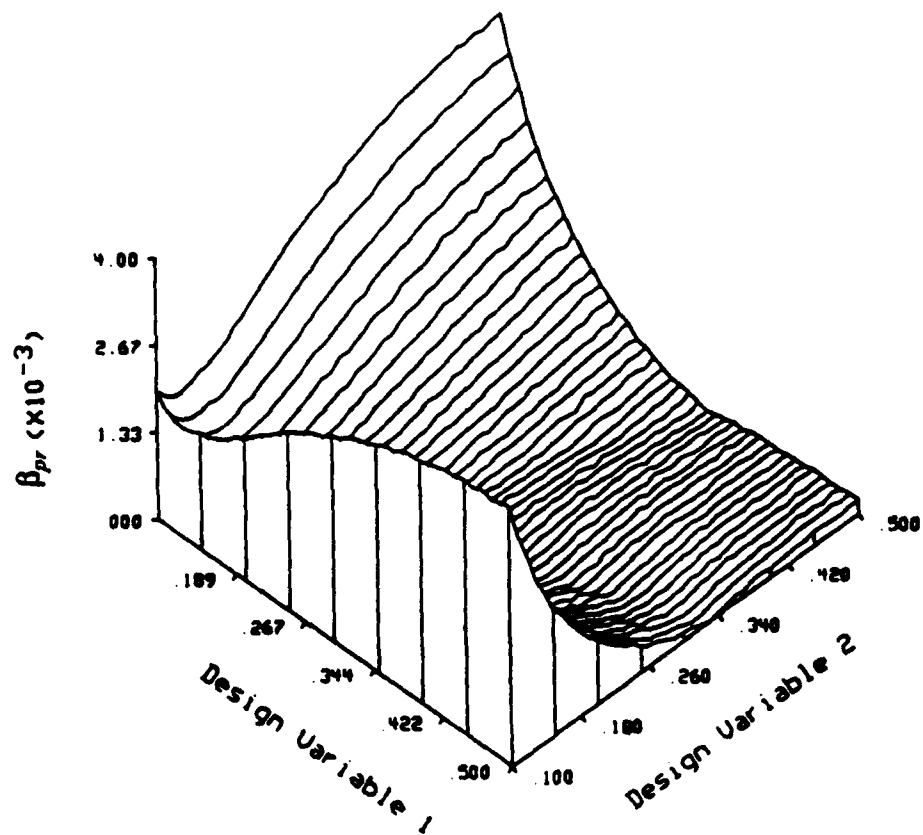


$m = 1.0$   
 $E = 10 \times 10^6$   
 Density = 4.6  
 Nominal areas = 0.1

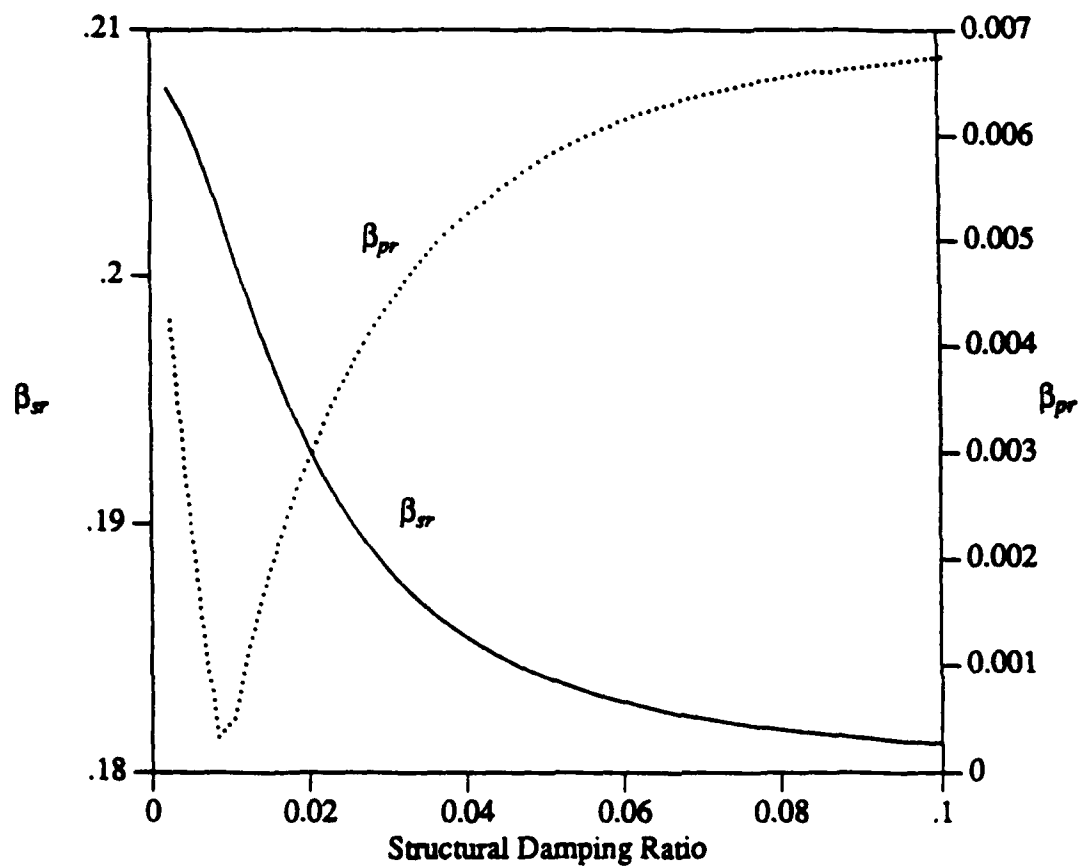
Figure 4.1 Two-Bar Truss



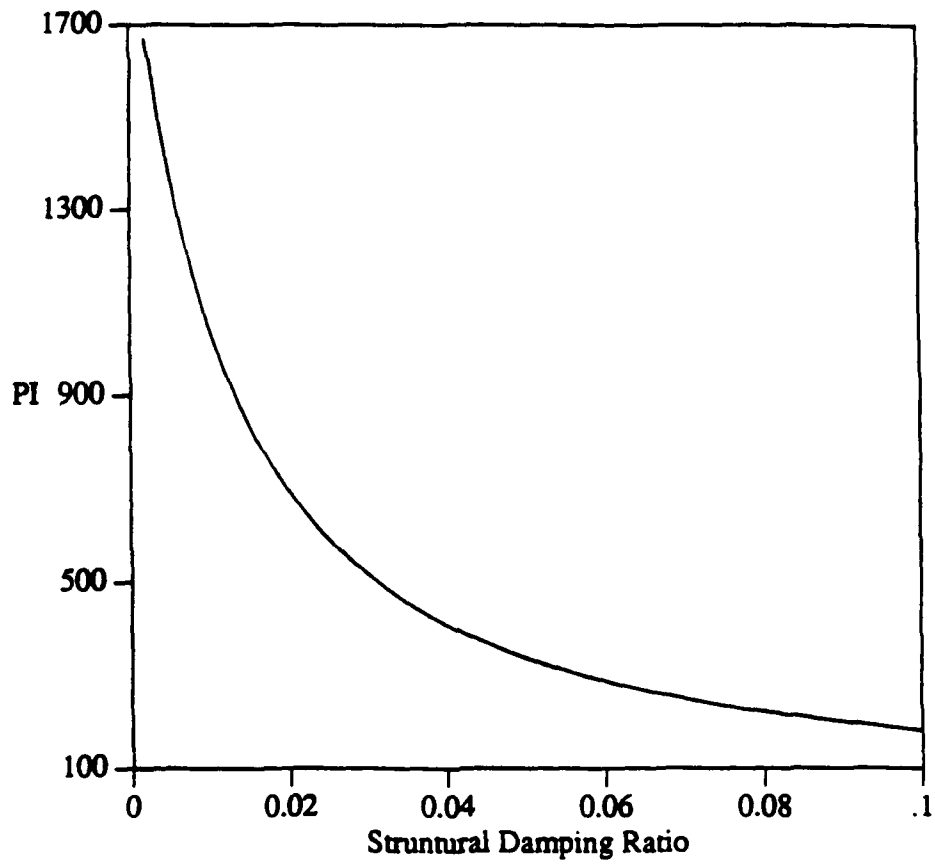
**Figure 4.2** Stability Robustness Index vs the Value of Design Variables for Two-Bar Truss



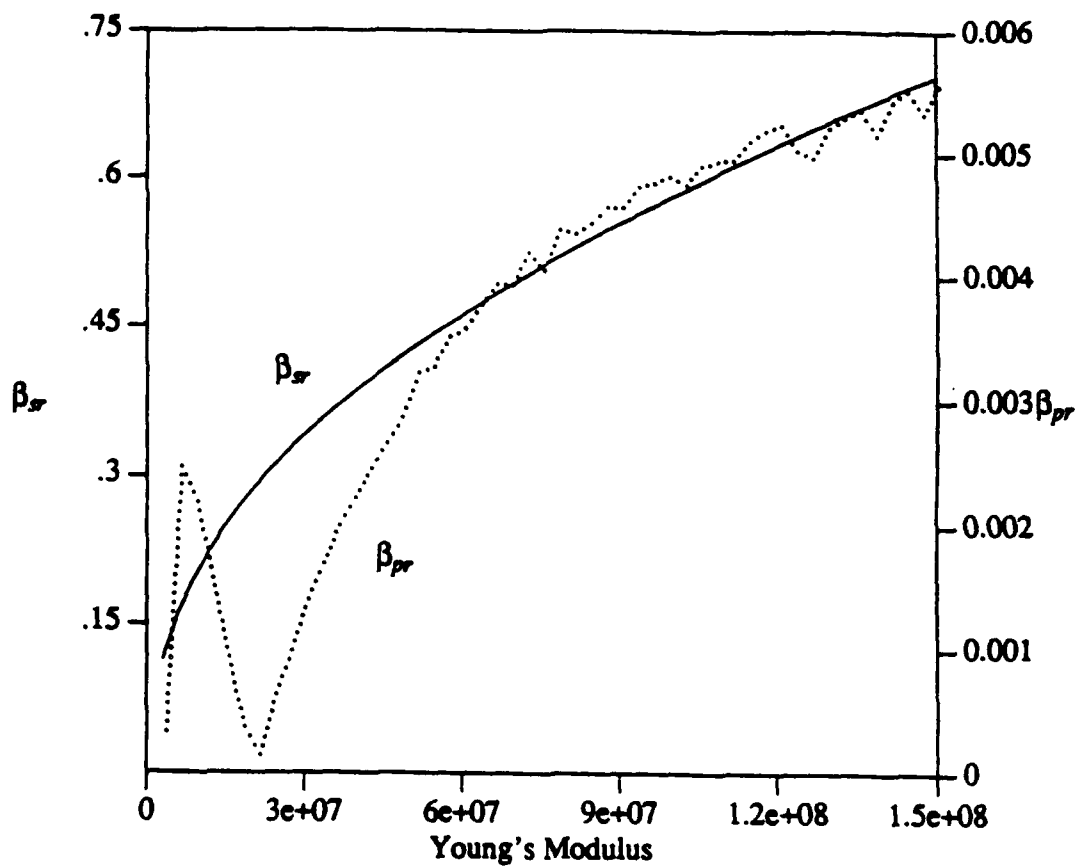
**Figure 4.3** Performance Robustness Index vs the Value of Design Variables for Two-Bar Truss



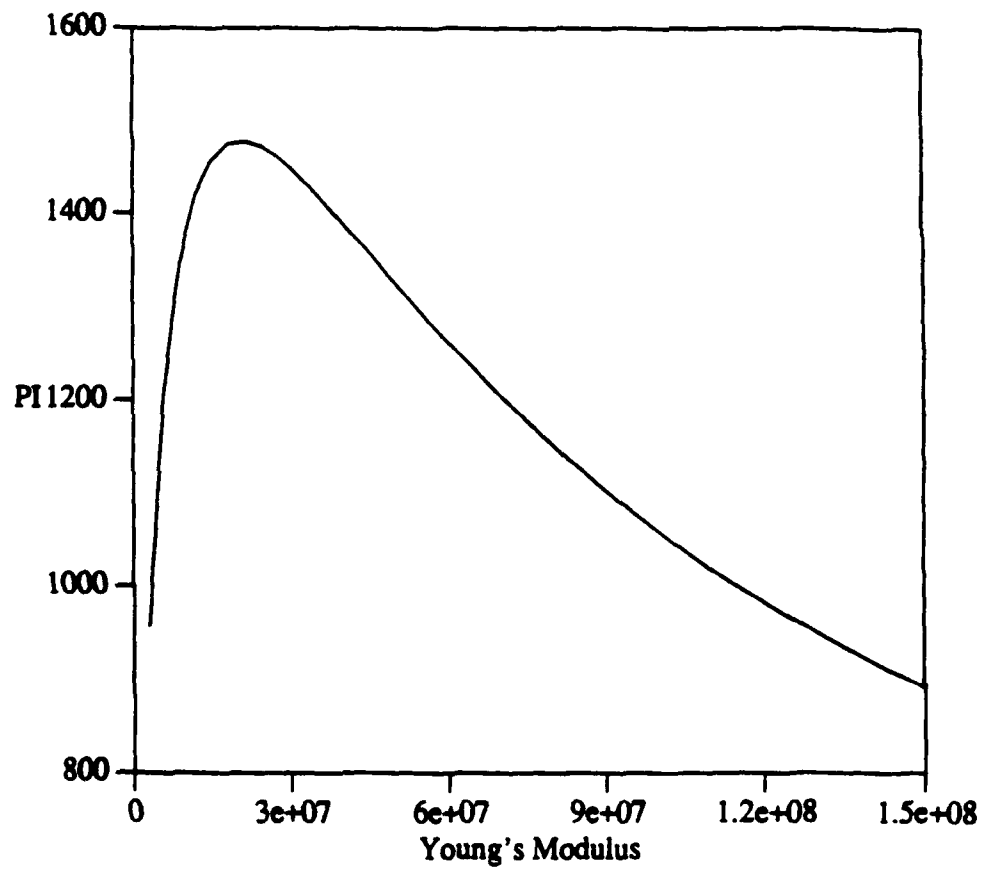
**Figure 4.4** Stability and Performance Robustness Indices vs the Structural Damping Ratio for Two-Bar Truss



**Figure 4.5** Performance Index vs the Structural Damping Ratio for Two-Bar Truss

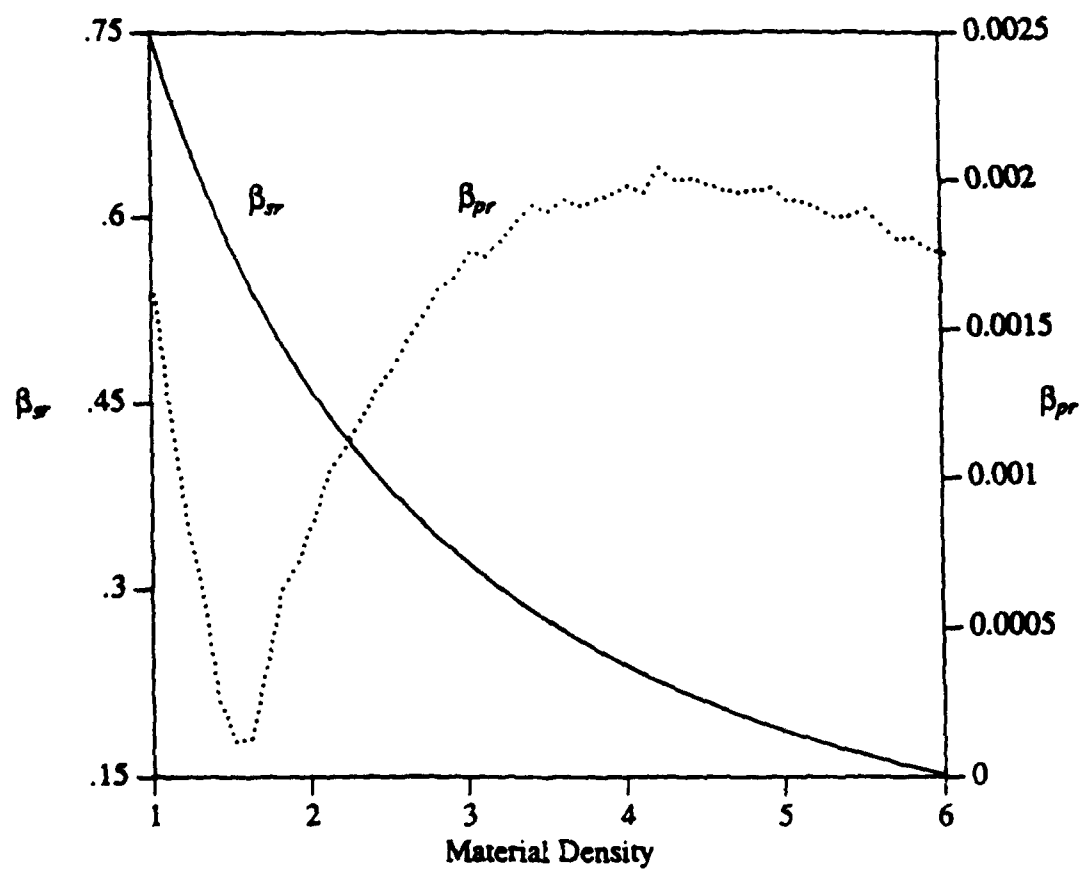


**Figure 4.6** Stability and Performance Indices vs the Young's Modulus for Two-Bar Truss

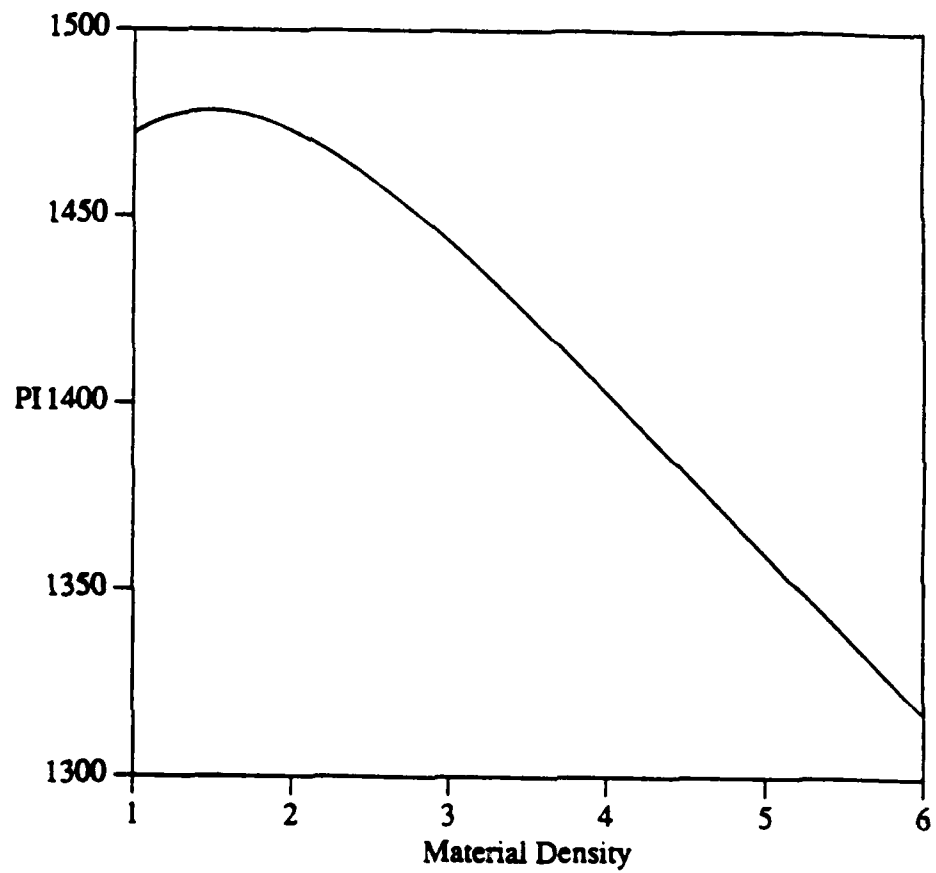


**Figure 4.7** Performance Index vs the Young's Modulus for Two-Bar Truss

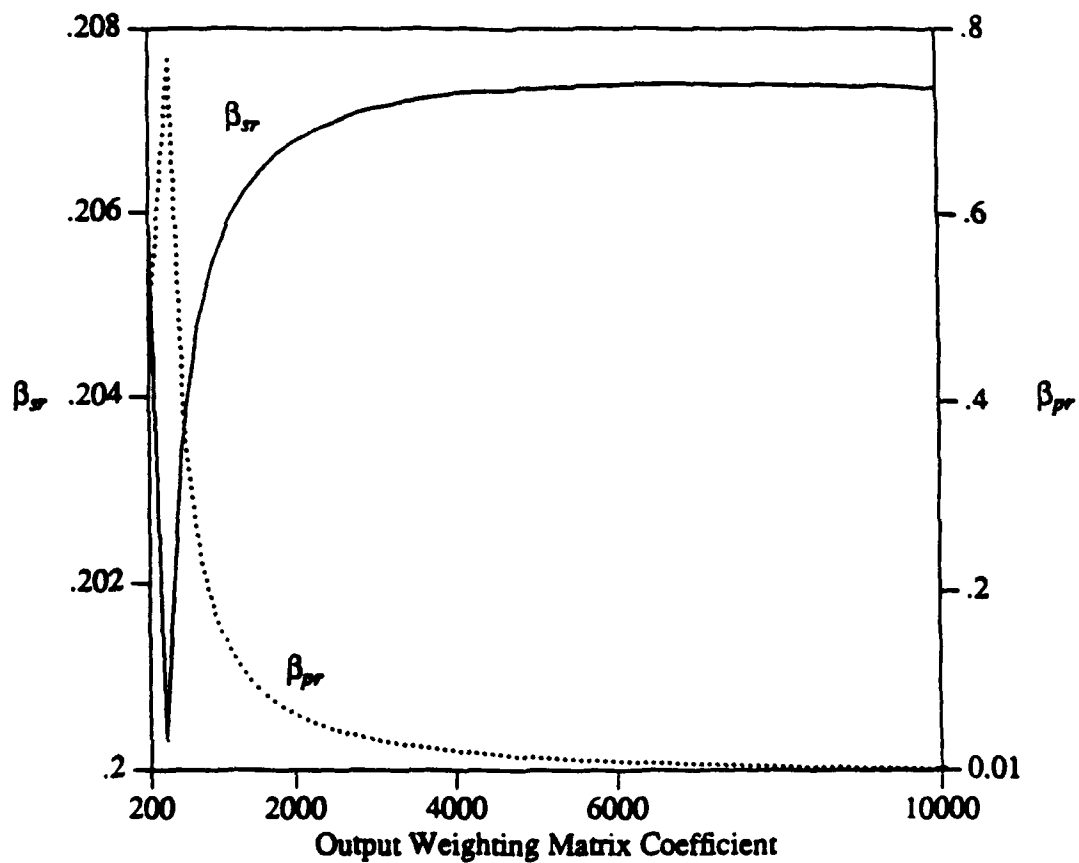




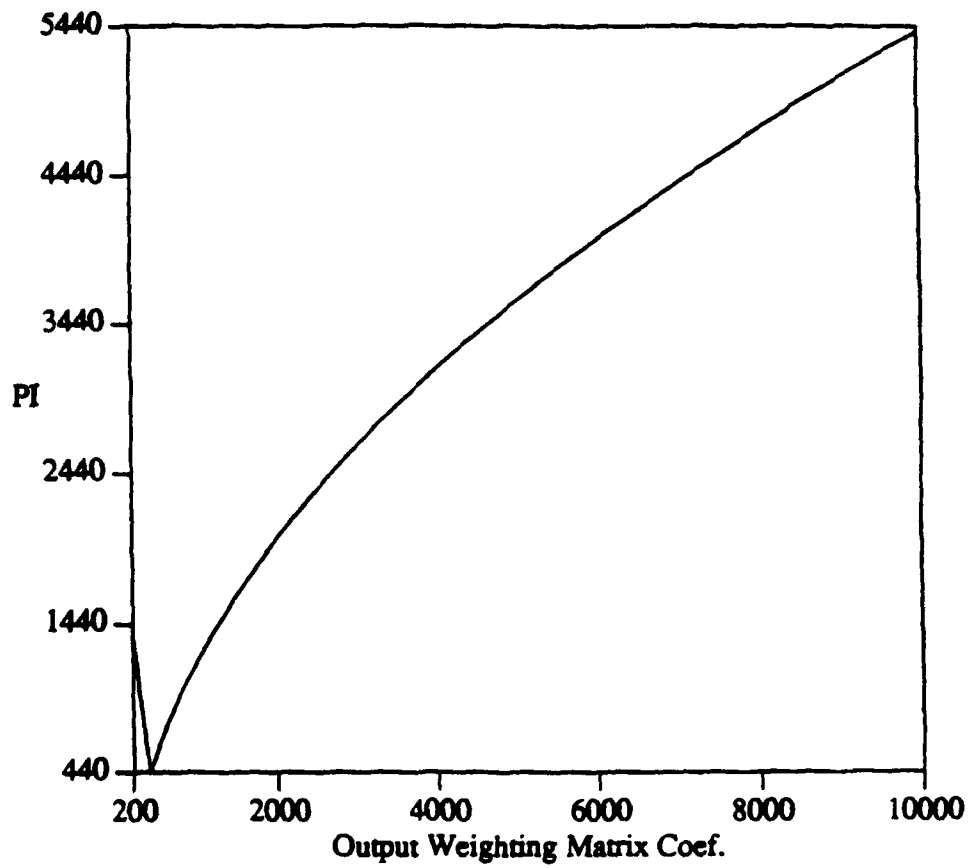
**Figure 4.8** Stability and Performance Robustness Indices vs the Material Density for Two-Bar Truss



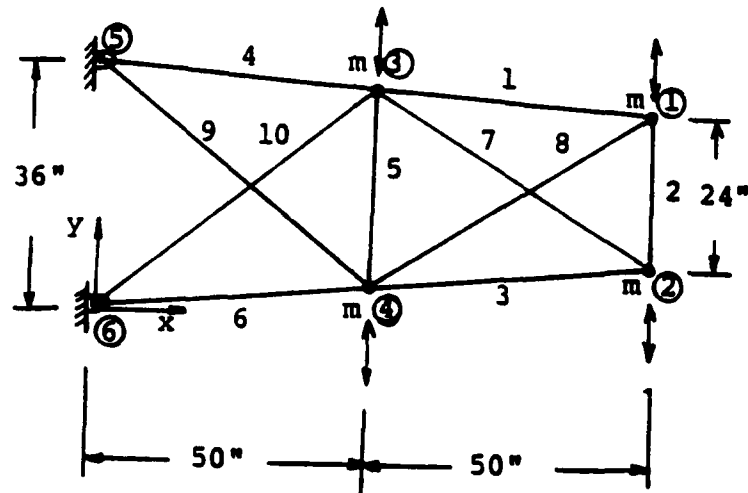
**Figure 4.9** Performance Index vs the Material Density for Two-Bar Truss



**Figure 4.10** Stability and Performance Robustness Indices vs the Output Weighting Matrix Coefficient for Two-Bar Truss

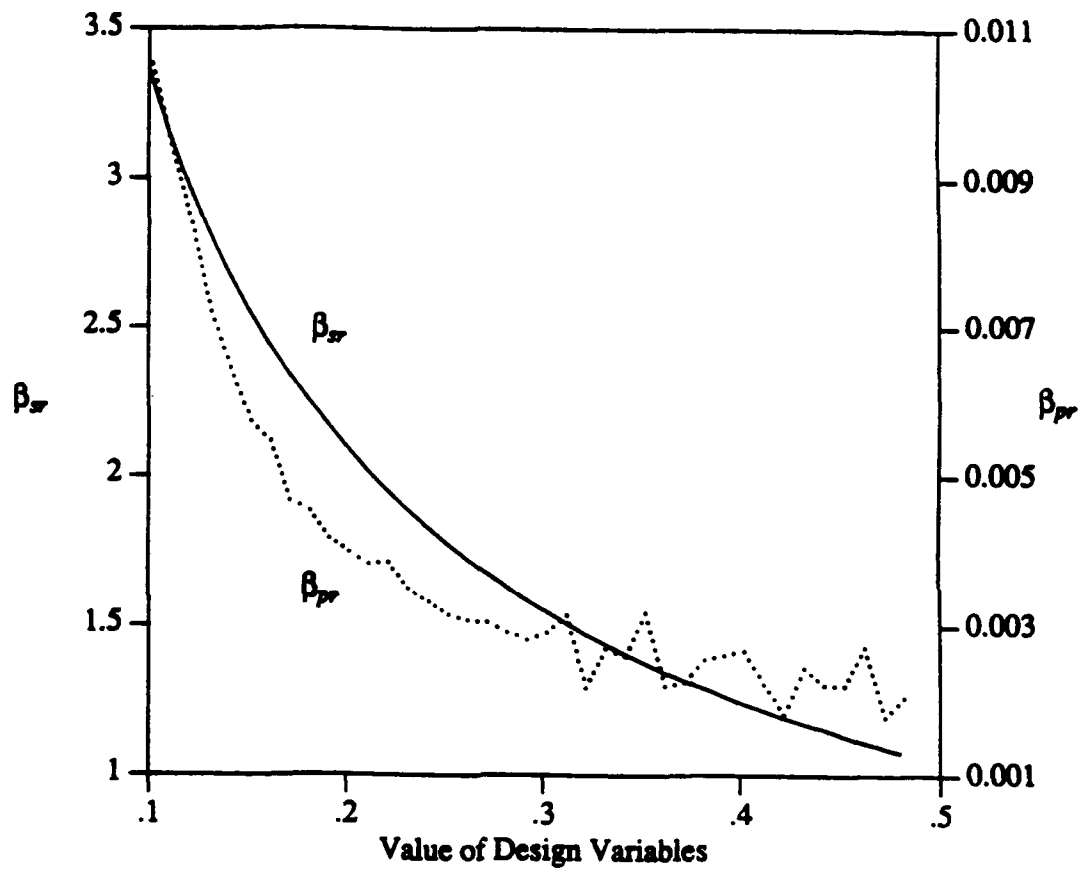


**Figure 4.11** Performance Index vs the Output Weighting Matrix Coefficient for Two-Bar Truss

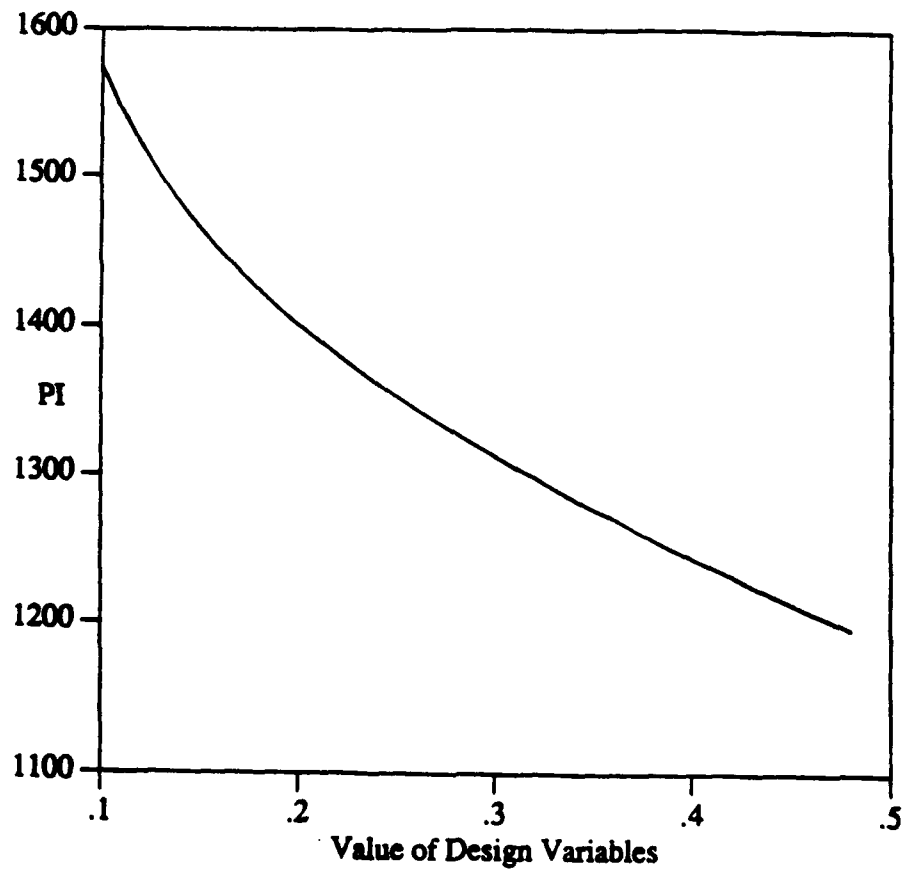


$m = 1.29$   
 $E = 10 \times 10^6$   
 Density = 0.1/32.2  
 Nominal areas = 0.1

Figure 4.12 Two-Bay Truss



**Figure 4.13** Stability and Performance Robustness Indices vs the Value of Design Variables for Two-Bay Truss



**Figure 4.14** Performance Index vs the Value of Design Variables for Two-Bay Truss

## CHAPTER 5

### ACTUATOR/SENSOR LOCATION SELECTION

The problem of selection of optimal locations of actuators/sensors in large flexible structures has been under study for more than a decade. This problem was considered as an integer programming problem and solved using heuristic techniques by Haftka and Adelman (1985), and Skelton and DeLorenzo (1983). The complexity of Skelton and DeLorenzo's algorithm for  $m$  actuators on  $n$  available positions is given by

$$C_{DS} = \frac{1}{2}(n+m)(n-m+1) \quad (5.1)$$

The complexities of WOBI (worst-out-best-in) and ESPS (exhaustive-single-point-substitution) algorithms, studied by Haftka and Adelman, can be expressed as

$$C_{WOBI} = n \times m \quad (5.2)$$

and

$$C_{ESPS} = m^2 \times (n-m) \quad (5.3)$$

All these three heuristic techniques yield only local optimal solutions with no guarantee about the global optimum.

Recently, Salama et al. (1987) considered a similar problem in the framework of combinatorial optimization and solved it using the simulated annealing heuristic algorithm. The complexity of this formulation for  $m$  actuators on  $n$  available positions can be expressed as

$$C_{COM} = C(n, m) = \frac{n!}{m!(n-m)!} \quad (5.4)$$

Generally speaking, the simulated annealing algorithm will converge to the global optimum, which corresponds to the lowest energy state, if an appropriate cooling temperature  $\theta_h$  is chosen. The selection of the temperature,  $\theta_h$ , is very critical in the simulated annealing algorithm for which there is no systematic approach. Hence a trial and error procedure is usually used to find the value of  $\theta_h$ .

In this work, three additional methods are investigated for solving the actuator and sensor location problem.

#### 5.1 Problem Formulation

The optimal actuator/sensor location selection problem, by nature, is a zero-one type combinatorial optimization problem. The system equations (2.1) and (2.2) will be changed if



the locations of actuators/sensors are changed. Hence, a more flexible modeling technique is necessary during the optimization process. For consistency, the optimal linear quadratic regulator is used for the control system design. The system equations and the control gain are expressed in Eqs (2.2) to (2.6). The input matrix  $B$  in system equation (2.2) can be expressed as

$$B = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix} \quad (5.5)$$

in physical coordinates, and as

$$B = \begin{bmatrix} 0 \\ \Phi D \end{bmatrix} \quad (5.6)$$

in modal space.

The inverse of the input weighting matrix  $R$  is assumed to be a diagonal matrix containing ones and zeros only with 1's corresponding to locations with actuators and 0's corresponding to those without actuators. Hence, the system equations will remain unchanged during the optimization process. Similarly, the output weighting matrix  $Q$  can be modified such that the system equations for the estimation part are unchanged during the optimization process for sensor location selection. For simplicity, the estimation part is neglected, and only the actuator location selection problem is considered in this work.

The objective function (criterion) proposed to be used in the actuator/sensor location selection problem is the energy dissipated by the active controller which can be written as

$$E_c = \frac{1}{2} \int_0^{\infty} \dot{q}^T D_c \dot{q} dt \quad (5.7)$$

where  $\dot{q}$  is the velocity vector and  $D_c$  is the induced damping matrix by the active controller. By letting

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (5.8)$$

$$B = \begin{bmatrix} 0 \\ \bar{B} \end{bmatrix} \quad (5.9)$$

$$\bar{D}_c = \begin{bmatrix} 0 & 0 \\ 0 & D_c \end{bmatrix} \quad (5.10a)$$

or

$$\bar{D}_c = \begin{bmatrix} 0 & 0 \\ 0 & -\bar{B}R^{-1}\bar{B}^T P_{22} \end{bmatrix} \quad (5.10b)$$

the dissipation energy can be rewritten as

$$E_c = \frac{1}{2} \int_0^\infty \left\{ \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix} \bar{D}_c \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \right\} dt \quad (5.11)$$

Applying the transition matrix of states in Eq. (5.11) gives

$$E_c = \frac{1}{2} \left\{ \begin{bmatrix} q_0^T & \dot{q}_0^T \end{bmatrix} \int_0^\infty e^{A_d^T t} \bar{D}_c e^{A_d t} dt \begin{bmatrix} q_0 \\ \dot{q}_0 \end{bmatrix} \right\} \quad (5.12)$$

which can be simplified as

$$E_c = \frac{1}{2} \left\{ \begin{bmatrix} q_0^T & \dot{q}_0^T \end{bmatrix} \bar{P} \begin{bmatrix} q_0 \\ \dot{q}_0 \end{bmatrix} \right\} \quad (5.13)$$

where

$$A_{cl} = A + BG = A - BR^{-1}B^T P \quad (5.14)$$

and  $\bar{P}$  is the solution of the Lyapunov equation

$$A_{cl}^T \bar{P} + \bar{P} A_{cl} = -\bar{D}_c \quad (5.15)$$

Equation (5.12) shows that the energy dissipation depends on the initial state which may not be available for practical problems. A simple way to eliminate the dependence on the initial state is to average it out by assuming the initial state  $\{q_0 \dot{q}_0\}$  to be a random variable uniformly distributed on the surface of the  $2n$ -dimensional unit sphere. Hence, an upper bound on the dissipation energy can be expressed as

$$\bar{E}_c = \frac{1}{2} \text{tr}(\bar{P}) \quad (5.16)$$

Let

$$R^{-1} = \text{diag} \left[ \bar{r}_1 \ \bar{r}_2 \ \cdots \ \bar{r}_n \right] \quad (5.17)$$

be the inverse of the input weighting matrix, with  $\bar{r}_i$  denoting a binary variable indicating the presence or absence of an actuator at position  $i$ . The zero-one optimization problem for the actuator/sensor location selection problem can then be expressed as follows:

$$\begin{aligned} &\text{Maximize } \bar{E}_c \\ &\left\{ \bar{r}_1 \ \bar{r}_2 \ \cdots \ \bar{r}_n \right\} \end{aligned} \quad (5.18)$$

subject to

$$\bar{r}_1 + \bar{r}_2 + \cdots + \bar{r}_n = m$$

$$\bar{r}_i \in (0, 1), \ i = 1, 2, \dots, n$$

## 5.2 Sequential-Best-Adding (SBA) Algorithm

Sequential-Best-Adding algorithm is a simple heuristic search procedure. As Skelton and DeLorenzo's WOBI and ESPS approaches, the sequential-best-adding method finds a local optimum. However, the complexity is reduced very much, especially when a small number of actuators ( $m$ ) are used in a large feasible space ( $n$ ).

The SBA algorithm can be described as follows:

1. Set  $i=1$ ,  $j=0$ , and  $m$ = specified number of actuators to be used.
2. Evaluate the objective function by enumerating all possible combinations with  $i$  actuators where  $j$  ( $j \leq i$ ) of them have been fixed from previous calculations in the feasible space.
3. Find the best combination, set  $i=i+1$  and  $j=j+1$ .
4. If  $i > m$  then stop and list the best combination as the solution, otherwise go to step (2).

This approach can be depicted in the form of a tree structure as shown in Fig. 5.1 in which only one branch has been explored. Hence, the complexity of the SBA algorithm can be expressed as

$$C_{SBA} = \frac{1}{2}m(2n-m+1) \quad (5.19)$$

For a large system, the parallel processing techniques can be used to improve the efficiency of the sequential-best-adding algorithm.

### 5.3 Penalty Function Method

One possible way to solve nonlinear integer programming problems is use a penalty function approach in which a penalty is included for non-integer solutions. This relaxation technique can also be used for the actuator/sensor location selection problem. The relaxed version of equation (5.18) gives

$$\begin{aligned} &\text{Maximize } \bar{E}_c + F_p \\ &\left\{ \bar{r}_1 \bar{r}_2 \cdots \bar{r}_n \right\} \end{aligned} \quad (5.20)$$

subject to

$$\bar{r}_1 + \bar{r}_2 + \cdots + \bar{r}_n = m$$

where  $F_p$  is the penalty function because of the non-zero-one design parameters.

One type of penalty function for general integer or discrete optimization problem (Rao, 1984) is

$$F_p = \gamma \sum_{i=1}^n \left\{ 4 \left[ \frac{x_i - y_i}{z_i - y_i} \right] \left[ 1 - \frac{x_i - y_i}{z_i - y_i} \right] \right\}^{\beta} \quad (5.21)$$

where  $x_i$ , ( $i=1, \dots, n$ ) are design parameters,  $\gamma$  and  $\beta$  ( $>1$ ) are constants and  $y_i$  and  $z_i$  are the two neighboring integer values of  $x_i$ :

$$y_i \leq x_i \leq z_i \quad (5.22)$$

For a zero-one programming problem,  $y_i$  and  $z_i$  are 0 and 1, respectively. Hence the penalty function in Eq. (5.21) can be simplified as

$$F_p = \gamma \sum_{i=1}^n \left\{ 4x_i(1-x_i) \right\}^{\beta} \quad (5.23)$$

This penalty function is shown in Fig. 5.2 for different values of  $\gamma$  and  $\beta$ . The maximum magnitude of the penalty function is changed when the value of  $\gamma$  is changed, while its shape is adjusted by the value of  $\beta$ . The choice of the value of  $\gamma$  strongly influences the convergence of the objective function while that of  $\beta$  does not. A large value of  $\gamma$  might cause to find only a local optimum. Hence, the value of  $\gamma$  is selected small enough at the beginning and increased gradually. The value of  $\beta$  is a constant and greater than 1 in order to maintain the continuity of the first derivative of the penalty function over boundary points.

Another form of penalty function is an exponential form:

$$F_p = \gamma \sum_{i=1}^n \left[ \left( 1 - e^{-\alpha x_i} \right) \left( 1 - e^{-\alpha(1-x_i)} \right) \right]^\beta \quad (5.24)$$

This function gives a flatter shape penalty function on non-feasible solutions as shown in Fig. 5.3. As with the penalty function of Eq.(5.23), the value of  $\gamma$  in Eq.(5.24) strongly affects the convergence of the optimization process. The values of  $\alpha$  and  $\beta$  influence the shape of the penalty function. The value for  $\alpha$  is suggested to be no greater than 10.

## 5.4 Genetic Algorithms

### 5.4.1 Background

Genetic algorithms (Davis, 1987 and Goldberg, 1989) are basically guided random search techniques derived from the natural genetics of populations. The decision variables (or design parameters) are coded as a string of binary bits which correspond to the chromosomes in natural genetics. The objective function value corresponding to the design vector plays the role of fitness in natural genetics. The artificial recombination among the population of strings is based on the fitness and the accumulated knowledge. In every new generation, a new set of strings is created by using randomized parents selection and crossovers from the old set of strings (or old generation). Although randomized, genetic algorithms are not simple random search techniques. They efficiently explore the new combinations with the available knowledge to find a new generation with better fitness or objective function value.

Genetic algorithms have been developed and studied by Holland (1973) and his students. These algorithms are computationally simple but powerful in their search for improvement. A growing number of applications can be found in science, engineering, business and social sciences.

In the area of optimization, three basic approaches are commonly used for solving nonlinear programming problems. The first category is calculus based methods which use the local gradient information to decide the search direction and to determine the convergence of optimum. These approaches find local optima instead of global ones for multimodal functions.

They are not useful for discrete or integer programming problems. The second category is enumerative schemes which are commonly used in discrete or combinatorial optimization. Although these methods are simple, they can only be applied to a problem with a small search space. For practical problems, the search space usually is too large to enumerate the entire space; instead, heuristic algorithms and knowledge based search are used to increase the search efficiency. The third category belongs to random search techniques which have attracted much attention when the previous two types of approaches fail. Random search can be divided into unguided and guided random searches. Unguided random searches can be expected to do no better than enumerative methods. The genetic algorithm, a guided random search, uses random selection as a tool to guide a highly efficient search through a binary coded feasible design space.

The genetic algorithm has been applied to different optimization problems by De Jong (1980) and Goldberg and Samtani (1986). It doesn't mean that genetic algorithm can find the optimal solution for all kinds of problems efficiently. On the other hand, genetic algorithms usually give reasonable near optimal solutions (a set of solutions). Hence, genetic algorithms are very useful to solve a difficult and complex problem which can not effectively be solved by conventional approaches. Moreover, since the whole process is manipulated on a set of points (population), genetic algorithms are suitable for parallel processing to increase the efficiency.

#### **5.4.2 Genetic Algorithms in Optimization**

A simple genetic algorithm involves copying strings and swapping partial strings between two mating strings. Three basic but very important operators - reproduction, crossover and mutation - used to produce new generations over and over again.

Reproduction is a randomized selection process in which individual strings are copied according to their fitness (or objective function) value. Similar to the natural selection process in which a Darwinian survival of the fittest among string creatures, the string with a higher fitness value has a higher probability to be reproduced. The probability of reproduction can be determined by dividing the individual fitness by the sum of fitnesses of the current generation. For example, if a set of strings with a population size 5 have fitnesses 50, 75, 100, 125, and 150, their probabilities of reproduction are  $50/500$ ,  $75/500$ ,  $100/500$ ,  $125/500$ , and  $150/500$ , respectively. Once a string is chosen for reproduction, a duplication is made and the replica is sent to the mating pool for further genetic operator actions.

Crossover (or recombination) is a primary operator in the mating process which generates the offsprings or new generations. Two steps are involved in the crossover process. The first step is randomly selecting the position to break (crossover) between two mating couple. The second step is reunion of these two mating couple. In natural genetic theory, the recombination fraction depends on the distance between the chromosomes of the mating couple. The closer the two loci are, the less likely that such a breakage and reunion will occur between them; the

farther apart they are, the more likely such a recombination will occur. For the simple artificial genetic algorithm, a fixed and preassigned probability of crossover is used. For example, let two strings  $A_1$  and  $A_2$  with 8-bit be selected and reproduced in the mating pool as:

$A_1$ : 01100100  
 $A_2$ : 10011100

Seven crossover positions are possible to select. If the crossover point is chosen at position 4 by a random choice, the two new strings  $A'_1$  and  $A'_2$  are generated to be a part of new generation as:

$A_1$ : 0110|0100       $A'_1$ : 01101100  
 $A_2$ : 1001|1100       $A'_2$ : 10010100

where | in  $A_1$  and  $A_2$  denotes the crossover position. This process can be implemented by (i) selecting the crossover position by a uniformly distributed random process between 1 and the string length minus 1, i.e.,  $[1, l-1]$  and (ii) swapping the contents of mating couple after the crossover point.

Mutation plays a secondary role in the operation of genetic algorithms. Mutation, changing a particular bit of coded string from 0 to 1 or vice versa, is a random walk through the string space. In natural genetics, mutation is the ultimate source of genetic variation which can arise only if there is some change in the genetic material. Because spontaneous mutation rates are typically quite small (on the order of  $10^{-4}$  to  $10^{-6}$  mutations per locus per generation), the effect is very small over the course of a few generations. On the other hand, the cumulative effects of mutation over a long period of time can become appreciable. Hence, mutation is needed in artificial genetic systems to act as an insurance policy against premature loss of important information.

The genetic algorithm can be applied for solving optimization problems as follows:

1. An appropriate chromosome representation should be defined to represent the combinations of design parameters which correspond to the fitness or objective function values. The representation should be one-to-one mapping in order to have a normal coding and decoding processes.
2. The population size and the maximum number of generations should be specified. The probabilities of crossover and mutation are selected. A set of initial population in the genetic system will also be generated.
3. Evaluate the fitness (or objective function) value of each individual in the current generation. The objective function plays the role of the environment to decide the fitness of a chromosome. Two important steps are usually taken to modify the objective function values for fitness evaluation. The first step is scaling to change the distribution range of the fitness value. The second one is normalization process which can change the speed of fixation of population. For this reason, users of genetic algorithms have employed a variety of normalization techniques. If normalization process stresses

improvements too much it will lead to a premature dominance of a single string. When this happens crossover becomes of little value, and the algorithm ends up in intensively searching the solution space in the region of the last good individuals found. If it doesn't stress good fitness, the algorithm might fail to converge to a good result and will be more likely to lose the best combination of its population. Hence, the normalization process is critical in genetic algorithms.

4. Apply the three operators - reproduction, crossover and mutation - on the old generation to generate the new population for the next generation.
5. Repeat step (3) and (4) until the maximum number of generations is achieved.

#### 5.4.3 Some Probability Relations in Genetic Algorithms

To understand why the genetic algorithms work, some probability study would be helpful. Three operators without restriction are considered in this section. A schema in genetic algorithms, chromosomes with specified 0-1 combination, is defined as the symbol set  $\{0,1,*\}$  in which \* denotes 'don't care'. To quantitatively evaluate the probability of survival of a specific schema, the schema order and defining length are needed. The order of a schema A,  $o(A)$ , is the number of fixed positions (i.e. 0's and 1's) present in the template. For example,  $A1=011*0***$ ,  $A2=1*1****1$  have schema orders  $o(A1)=4$  and  $o(A2)=3$ , respectively. The defining length of a schema A,  $\delta(A)$ , is the distance between the first and the last specific string positions. For example, A1 and A2 have the defining lengths  $\delta(A1)=5-1=4$ , and  $\delta(A2)=8-1=7$ , respectively.

During reproduction, a string is selected with the probability of its own fitness divided by the total fitness in that generation. After picking n individuals from the population which have the the probability  $P(A,i)$  in which A indicates the specific schema and i denotes the  $i^{th}$  generation,  $P(A,i+1)$  can be expressed as follows:

$$P(A,i+1) = P(A,i) \times n \frac{f(A)}{\sum_i f_i} \quad (5.25a)$$

or

$$P(A,i+1) = P(A,i) \frac{f(A)}{\bar{f}} \quad (5.25b)$$

where  $f(A)$  and  $\bar{f}$  denote the fitness of specific schema and average fitness, respectively.

During crossover, a schema can survive if the crossover site falls outside the defining length. Hence the survival probability under simple crossover is given by



$$P_s = 1 - \frac{\delta(A)}{l-1} \quad (5.26)$$

where  $l$  is the length of the string  $A$ . If a random choice is used in crossover and the probability of crossover is selected as  $P_c$ , the survival probability is given by the expression

$$P_s \geq 1 - P_c \frac{\delta(A)}{l-1} \quad (5.27)$$

Mutation is the random alternation of a single position with a specified probability  $P_m$ . A specific schema  $A$  can survive if and only if all alleles (elements) at the specified positions must themselves survive. A single allele survives with a probability of  $1 - P_m$  and hence the survival probability of schema  $A$  because of mutation is

$$P_{sm} = (1 - P_m)^{o(A)} \quad (5.28)$$

Since  $P_m$  is usually very small compared to 1, Eq. (5.28) can be simplified as

$$P_{sm} = (1 - o(A) \times P_m) \quad (5.29)$$

Therefore, the probability of a specific schema in the next generation, under three genetic operators, can be obtained by combining equations (5.25), (5.27), and (5.29):

$$P(A, i+1) \geq P(A, i) \frac{f(A)}{\bar{f}} \left[ 1 - P_c \frac{\delta(A)}{l-1} \right] \left[ 1 - o(A) \times P_m \right] \quad (5.30)$$

or

$$P(A, i+1) \geq P(A, i) \frac{f(A)}{\bar{f}} \left[ 1 - P_c \frac{\delta(A)}{l-1} - o(A) \times P_m \right] \quad (5.31)$$

Since  $P_m$  is very small compared to 1 or  $P_c$ , and 1,  $\bar{f}(A)$ ,  $P_c$ , and  $P_m$  are constants,  $\delta(A)$  becomes much more important than  $o(A)$ . Hence, the fitness and the defining length of string  $A$  become the two factors to decide the survival probability. An observation of Eq.(5.31) shows that the short, low-order, and above-average schemata receive an exponentially increasing probability in subsequent generations.

#### 5.4.4 More on Simple Genetic Algorithms

For the actuator/sensor location selection problem, a type of combinatorial optimization problem, the genetic algorithm will be a suitable approach. The chromosomal representation of

the design vector simply involves using a string of binary bits. The locations in the binary string indicate the numbered positions on the controlled structure to place the actuators or sensors. The 1's and 0's in the binary string denote the presence and absence of the actuators/sensors. For example, the string 10100100 means that three actuators (or sensors) are chosen and located at positions 1,3, and 6 out of the possible 8 positions.

Several modifications are made to the genetic algorithm described in section 5.4.2 before using it for the solution of the actuator/sensor location selection problem:

1. The initial population is randomly generated with the restriction that no two individuals be allowed to have the same chromosome in order to get the maximal variety.
2. The successive flips of a biased coin were used to generate the initial population efficiently.
3. A normalization and scaling process has been applied to increase the differences between the fitnesses of different chromosomes. The normalization process is indicated by the relation:

$$f'(x) = \left[ 1 - \frac{f_{\max} - f(x)}{f_{\max} - f_{\min}} \right]^2 \quad (5.32)$$

where  $f(x)$  is the objective function value with design vector  $x$  and  $f'(x)$  is the normalized fitness value for chromosome  $x$ .

4. A constraint is placed on the total number of actuators used. Two approaches can be used for this purpose. In the first approach, a restriction is placed on the crossover process which is permitted to generate two children with the specified number of actuators (i.e., number of 1's in the chromosome). Otherwise, the crossover process is not allowed. In the second approach, no restriction is placed on the crossover process; but a corrective action is taken afterwards. After mating, the number of 1's in the two children are made equal to the specified number by a random mutation process. For example, let two mating couple with chromosomes be given by

$A_1: 10100010$

$A_2: 00101010$

where the total number of 1's is specified as 3. According to the first method, the only possible positions for crossover are 5 (i.e., the position between 5<sup>th</sup> and 6<sup>th</sup> alleles from left side), 6, and 7; and none of them gives a new chromosome (or combination). On the other hand, no more improvement can be made through these two mating couple.

If the second approach is chosen, crossover can be made at any position among 1 to 7 with a probability  $\frac{1}{7}$ . If, for example, position 4 is randomly chosen for crossover, then

$$\begin{array}{lcl}
 A_1: 10100010 & P = \frac{1}{7} & A'_1: 10101010 \\
 A_2: 00101010 & & A'_2: 00100010
 \end{array} \quad (5.33)$$

The numbers of 1's in  $A'_1$  and  $A'_2$  do not satisfy the constraint (value of 3) and hence a random mutation should be used on  $A'_1$  ( $A'_2$ ) to reduce a 1 in  $A'_1$  (increase a 1 in  $A'_2$ ). Suppose the optimal combination is "10101000", then the probability of generating the optimal offspring is  $\frac{1}{28}$ .

$$\begin{array}{lcl}
 A'_1: 10101010 & \xrightarrow{P_1 = \frac{1}{4}} & 10101000 \\
 A'_2: 00100010 & \xrightarrow{P_2 = \frac{1}{6}} & 10100010
 \end{array} \quad (5.34)$$

Combining Eqs (5.33) and (5.34) gives

$$\begin{array}{lcl}
 A_1: 10100010 & \xrightarrow{P \times P_1 = \frac{1}{28}} & 10101000 \\
 A_2: 00101010 & \xrightarrow{P \times P_2 = \frac{1}{42}} & 10100010
 \end{array} \quad (5.35)$$

It is obvious that the second method gives more freedom in the crossover process and will reduce the probability of non-improving crossover.

5. A variable population size is used in the actuator/sensor location selection problem. Since the genetic algorithm is highly dependent on the normalization technique used, the best chromosome which was found since the beginning of the process is not guaranteed to be included in the current generation. To preserve the best individual, the best individual in the old generation (A) is duplicated to new generation if there is no other individual who is at least as good as A. Hence, the population size is increased by 1 if a duplication is made.
6. Although random mating is the most important mating system in many natural populations, there are certain departures from the random mating that can also be very important. The definitions and characteristics of several important mating systems are listed below:

*Random mating:* Choice of mates independent of genotype and phenotype.

*Positive assortative mating:* Mates phenotypically more similar than would be expected

by chance.

*Negative assortative mating:* Mates phenotypically more dissimilar than would be expected by chance.

*Inbreeding:* Mating between relatives.

In humans, positive assortative mating occurs for height, IQ score and certain other traits. Negative assortative mating is apparently quite rare, but an obvious example is sex. The principal effect of inbreeding in a population is to increase the frequency of homozygous genotypes at the expense of the frequency of heterozygous genotypes. This effect can most easily be seen in the case of repeated self-fertilization. Detrimental effects of inbreeding (called inbreeding depression) are found in virtually all outcrossing species, and the more intense the breeding, the more harmful the effects. The inbreeding might happen in random mating process with no restriction. The frequency (or probability) of inbreeding will increase when the population with the same genotype increases. In artificial genetic algorithms, the inbreeding process gives no new information and reduces the time to fixation. Hence, the restriction on inbreeding should be considered.

## 5.5 Examples

To illustrate the feasibility of the three methods discussed in the previous sections, a two-bay truss (Fig. 2.3) and a six-bay truss (Fig. 2.9) are considered as examples. The number of actuators is assumed to be 3. The optimal linear quadratic regulator is applied to solve the optimal control gain. The dissipation energy of active controller is used as the objective function for maximization. The output weighting matrix is assumed to be  $1000 \cdot I$  ( $I$  is the identity matrix) in both the examples. The input weighting matrix has fixed dimension of  $n$  ( $n$  is the total number of d.o.f.'s in the structure) and is composed as described in section 5.1.

By using the sequential-best-adding algorithm, the result of the two-bay truss is presented in Table 5.1. The optimal solution for level 1 (i.e., when one actuator is used) is at position 2. The optimal solution for level 2 (when two actuators are used) is at positions 2 and 4 which is a child in branch 2. But the optimal solution for level 3 (when three actuators are used) is at positions 2, 4, and 7 which is not same as the global optimal solution (1,3,5). The number of function evaluations used is 21 out of a total combination of 56 ( $c(8,3)=8!/3!5!$ ). For the six-bay truss, the global optimal solution for 3 actuators is at (2,3,4). The SBA approach gives the optimal solutions for levels 1 to 3 at (2), (2,4), and (2,3,4), respectively. The number of function evaluations used is 69 ( $=24+23+22$ ) out of a total combination of 2024 ( $c(24,3)=24!/21!3!$ ).

The optimal selections of actuator locations for the two-bay truss using the penalty function method with different parameters are shown in Table 5.2. None of the parameters give the global optimal solution (1,3,5). Also, the number of function

evaluations is much higher than the total number of combinations (56). Hence, this is not a feasible approach for the actuator/sensor location selection problem.

The genetic algorithm is also applied to the same examples. For the two-bay truss, the initial population size and the maximum number of generations are selected as 8 and 15, respectively. Results obtained with different crossover probability (from 0.6 to 1.0) and mutation probability (from 0.001 to 0.005) are shown in Table 5.3. The number of function evaluations in 10 generations varies from 18 to 35. Most of them find the global optimal solution (1,3,5). A small number of them find the second optimal solution (1,3,7) which is in the same branch (1,3) of the global optimal solution. Three randomly selected initial generations are used for comparison in two cases: (i)  $P_c=0.7$  and  $P_m=0.001$ , (ii)  $P_c=0.8$  and  $P_m=0.001$ . The results are shown in Figs. 5.4 to 5.6 for case (i) and in Figs. 5.7 to 5.9 for case (ii). Figure 5.4 shows that the total number of genotypes (configurations) generated is constant after generation 32, i.e., no new genotypes are generated. Figures 5.5 and 5.6 show that the average and the maximal fitnesses increase with the number of generations until the fixation occurs. The same trend is revealed in Figs. 5.7 to 5.9 also.

For the six-bay truss, the initial population size and the maximum number of generations are chosen as 20 and 40, respectively. The results obtained with different crossover and mutation probabilities are shown in Table 5.4. The number of function evaluations in 15 generations varies from 110 to 271. Most of them find the global optimal solution with the actuators at positions (2,3,4). Few of them find the second optimal solution at (1,2,4) which is in the same branch (2,4) of the global optimal solution. Three randomly selected initial generations are used for comparison in two cases: (i)  $P_c=0.7$  and  $P_m=0.001$ , and (ii)  $P_c=0.8$  and  $P_m=0.001$ . The relations between the total number of genotypes, average fitness, maximal fitness and the number of generations are shown in Figs. 5.10 to 5.12 for case (i) and in Figs. 5.13 to 5.15 for case (ii). The premature phenomenon occurs in case (i) (line c in Figs. 5.10 to 5.12). The only way to reduce the possibility of prematurity is to reduce the probability of inbreeding.

## 5.6 Concluding Remarks

Three approaches are presented for solving the actuator/sensor location selection problem. The sequential-best-adding method, although no global optimal solution is guaranteed, reduces the computational effort to a large extent. The complexity is proportional only to the number of variables involved in the optimization problem. For the two-bay truss, SBA gives local optimal solution which is the third global optimal solution, while SBA gives the global optimum for the six-bay truss. For a large flexible structure with a small number of actuators or sensors, the SBA approach will be very efficient.

The penalty function method complicates the zero-one actuator/sensor location selection problem and cannot converge to the best solution or local optimum.

The genetic algorithm, although a simple guided random search algorithm, is very effective in solving actuator/sensor location selection problem when posed as a zero-one programming problem. In most cases of the examples considered, the global optimal solution is found in reasonable number of function evaluations. The following recommendations can be made for the genetic algorithm (GA):

1. The size of population should be close to the length of the chromosome used in the genetic algorithm.
2. The number of generations needed in GA, in general, is a state of art. An interactive procedure to decide the termination of GA will be more profitable.
3. The probability of crossover should be high and the that of mutation should be low. The suggested values for  $P_c$  and  $P_m$  are 0.7 to 0.8 and 0.001 to 0.005, respectively.
4. No specific convergence criterion has been used in GA. When the best individual with the maximal fitness remains unchanged for several generations, say 5 or 10, we can assume that the process has converged.
5. As shown in the examples, GA gives very good results. However, in general, GAs do not guarantee convergence to the global optimal solution. A hybrid scheme is suggested for solving the actuator/sensor location selection problem. The hybrid procedure to be followed as: (i) start with the GA and find the best individuals in the last generation, (ii) explore the branches found in (i) as in SBA approach, and (iii) choose the best local optimum from step (ii).
6. To improve the efficiency of the procedure, parallel processing techniques can be used in genetic algorithms.

Table 5.1 Objective Function Values for Two-Bay Truss

Level 1 ( $\times 10^3$ )	Level 2 ( $\times 10^3$ )	Level 3 ( $\times 10^3$ )					
(1) 2.1469**	(1,2) 1.9535	(1,2,3) 1.9493	(1,2,4) 1.8965	(1,2,5) 1.8544	(1,2,6) 1.7293	(1,2,7) 1.7650	(1,2,8) 1.7158
	(1,3) 2.3262	(1,3,4) 1.9409	(1,3,5) 2.1149	(1,3,6) 1.9114	(1,3,7) 2.0654	(1,3,8) 1.9207	
	(1,4) 1.9370	(1,4,5) 1.9085	(1,4,6) 1.6565	(1,4,7) 1.7570	(1,4,8) 1.6512		
	(1,5) 2.1020	(1,5,6) 1.6599	(1,5,7) 1.8428	(1,5,8) 1.6988			
	(1,6) 1.8417	(1,6,7) 1.6166	(1,6,8) 1.8452				
	(1,7) 1.9694	(1,7,8) 1.6402					
	(1,8) 1.8973						
(2) 6.7855	(2,3) 1.9310	(2,3,4) 1.8899	(2,3,5) 1.7509	(2,3,6) 1.6888	(2,3,7) 1.8455	(2,3,8) 1.6663	
	(2,4) 3.2917	(2,4,5) 1.9393	(2,4,6) 1.7231	(2,4,7) 1.9642	(2,4,8) 1.7125		
	(2,5) 1.8886	(2,5,6) 1.5272	(2,5,7) 1.6079	(2,5,8) 1.5622			
	(2,6) 1.8492	(2,6,7) 1.5111	(2,6,8) 1.7079				
	(2,7) 1.9799	(2,7,8) 1.5303					
	(2,8) 1.8528						
(3) 2.1173	(3,4) 1.9503	(3,4,5) 1.8044	(3,4,6) 1.6915	(3,4,7) 1.8440	(3,4,8) 1.6748		
	(3,5) 2.0610	(3,5,6) 1.6169	(3,5,7) 1.8995	(3,5,8) 1.6543			

Note: \* - Actuator locations

\*\* - The value of objective function

Table 5.1 Objective Function Values for Two-Bay Truss (Continued)

Level 1 (*10 <sup>5</sup> )	Level 2 (*10 <sup>5</sup> )	Level 3 (*10 <sup>5</sup> )					
(3) 2.1173	(3,6) 1.8599	(3,6,7) 1.6556	(3,6,8) 1.8320				
	(3,7) 2.1180	(3,7,8) 1.6832					
	(3,8) 1.8395						
(4) 4.0737	(4,5) 1.9728	(4,5,6) 1.5046	(4,5,7) 1.6698	(4,5,8) 1.5336			
	(4,6) 1.8051	(4,6,7) 1.5231	(4,6,8) 1.6609				
	(4,7) 1.9847	(4,7,8) 1.5365					
	(4,8) 1.7925						
(5) 2.1772	(5,6) 1.6224	(5,6,7) 1.4662	(5,6,8) 1.6449				
	(5,7) 1.7893	(5,7,8) 1.5551					
	(5,8) 1.7115						
(6) 6.0217	(6,7) 1.6050	(6,7,8) 1.6324					
	(6,8) 2.3064						
(7) 2.2810	(7,8) 1.6750						
(8) 6.1063							



**Table 5.2    Penalty Function Method for Two-Bay Truss**

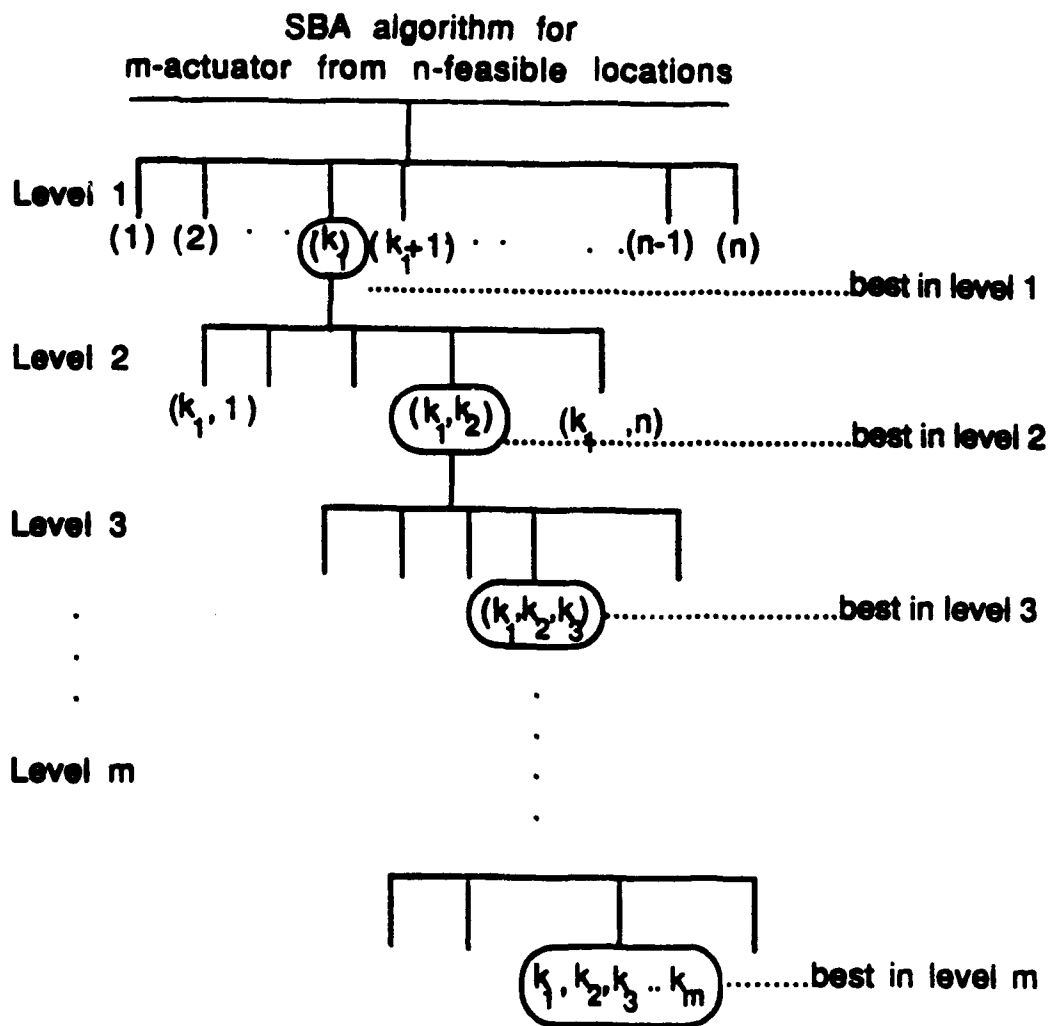
	Penalty function 1				Penalty function 2		
$\gamma$	0.2	0.02	0.0512	0.131	0.2	0.819	0.32
$\beta$	2.2	2.2	1.408	2.2	2.2	2.2	1.76
$x_i$	0	0	0	0	1	0	0
	0	0	0	0	0	0	1
	0	0	0	0	0	0	0
	1	1	0	0	1	1	0
	0	0	0	0	0	0	1
	0	1	1	1	0	1	0
	1	0	1	1	0	0	0
	1	1	1	1	1	1	1
$f(x)$ (*10 <sup>5</sup> )	1.54	1.662	1.633	1.633	1.651	1.661	1.562
number of eva- luations	443	292	359	367	369	363	364

**Table 5.3 Genetic Algorithm for Two-Bay Truss**

$P_c$	$P_m$	Best final genotype	Total # of evaluations	Obj. function	max. fitness found
1.0	0.001	10101000*	28	2.1149e+5	1.0
	0.002	10101000	22	2.1149e+5	1.0
	0.005	10101000	31	2.1149e+5	1.0
0.9	0.001	10101000	35	2.1149e+5	1.0
	0.002	10101000	35	2.1149e+5	1.0
	0.005	10101000	35	2.1149e+5	1.0
0.8	0.001	10100010**	27	2.0654e+5	0.9335
	0.002	10100010	28	2.0654e+5	0.9335
	0.005	10101000	32	2.1149e+5	1.0
0.7	0.001	10101000	25	2.1149e+5	1.0
	0.002	10101000	26	2.1149e+5	1.0
	0.005	10100010	32	2.0654e+5	0.9335
0.6	0.001	10101000	23	2.1149e+5	1.0
	0.002	10000101***	18	1.8452e+5	0.6659
	0.005	10101000	34	2.1149e+5	1.0
<p><b>Note:</b> * -- global optimum, ** -- second global optimum            *** -- local optimum            Population size = 8            Number of generations = 15            Total number of genotypes (combinations) = 56</p>					

**Table 5.4 Genetic Algorithm for Six-Bay Truss**

$P_c$	$P_m$	Best final genotype	Total # of evaluations	Obj. function	max. fitness found
1.0	0.001	genotype I*	189	9.372e+4	1.0
	0.002	I	269	9.372e+4	1.0
	0.005	I	271	9.372e+4	1.0
0.9	0.001	I	203	9.372e+4	1.0
	0.002	genotype II**	202	9.204e+4	0.9586
	0.005	I	271	9.372e+4	1.0
0.8	0.001	I	142	9.372e+4	1.0
	0.002	I	125	9.372e+4	1.0
	0.005	I	245	9.372e+4	1.0
0.7	0.001	II	110	9.204e+4	0.9586
	0.002	I	141	9.372e+4	1.0
	0.005	II	194	9.204e+4	0.9586
0.6	0.001	I	177	9.372e+4	1.0
	0.002	I	205	9.372e+4	1.0
	0.005	I	198	9.372e+4	1.0
<p>Note: * genotype I = 0111000000000000000000, global optimum  ** genotype II = 101100000000000000000000, second optimum  Population size = 20  Number of generations = 40  Total number of genotypes (combinations) = 2024</p>					



**Figure 5.1 Sequential-Best-Adding Algorithm Search Tree Structure**

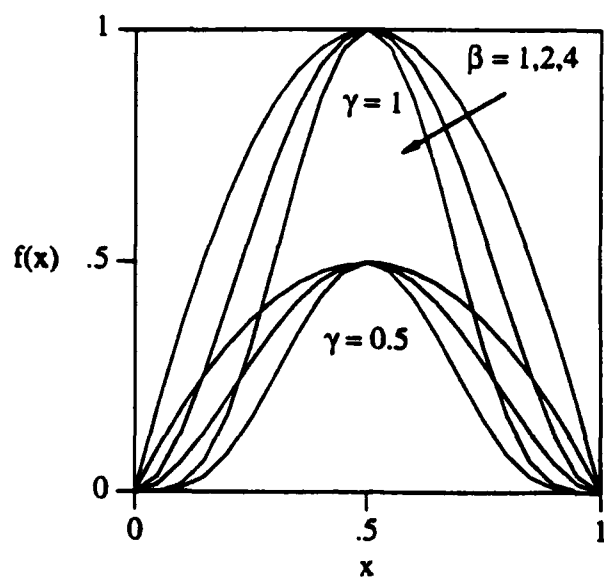


Figure 5.2 Penalty Function 1

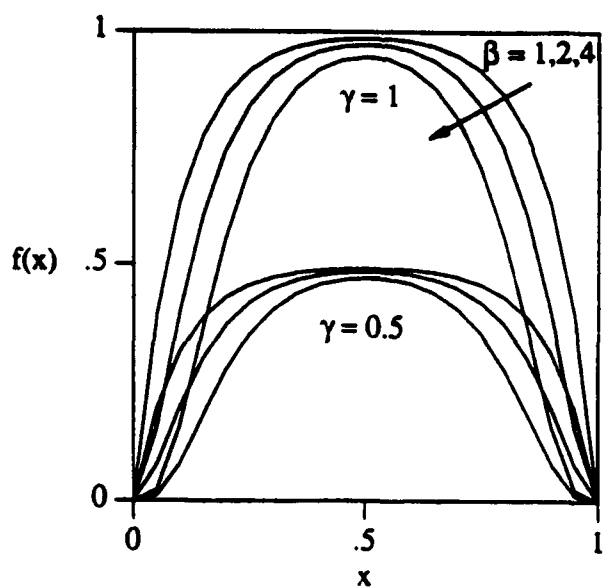
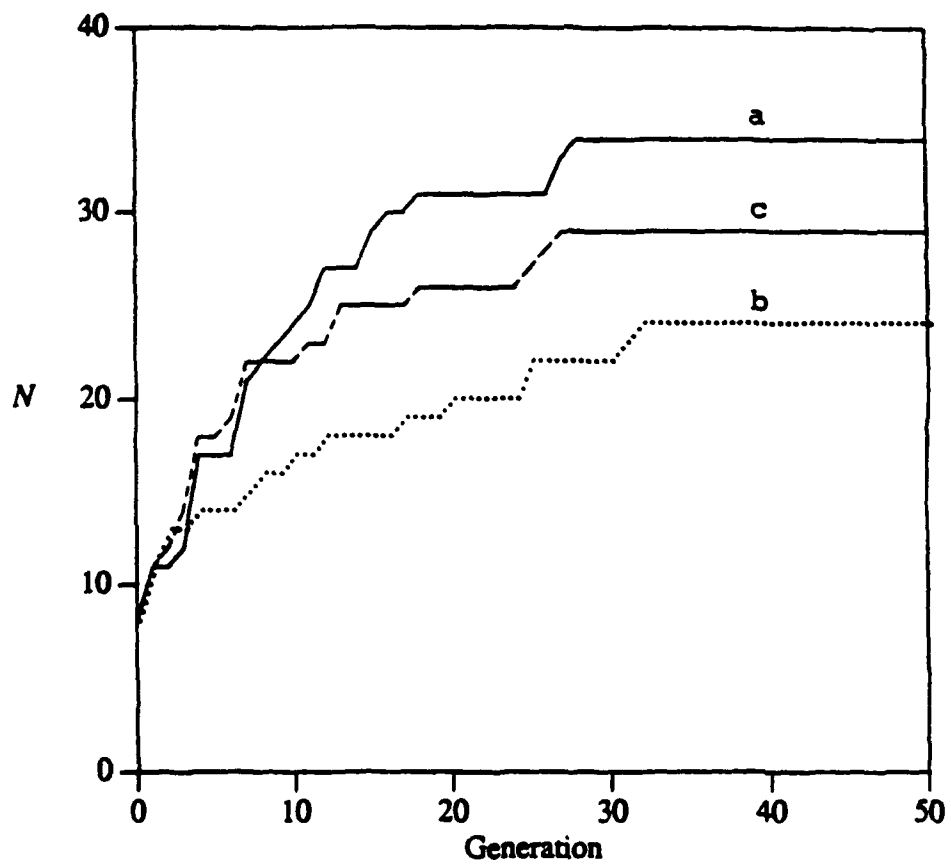


Figure 5.3 Penalty Function 2 ( $\alpha = 10$ )



**Figure 5.4** Accumulated Genotypes (Evaluations) for 2-Bay Truss with Different I.C.'s ( $P_c=0.7$ ,  $P_m=0.001$ )

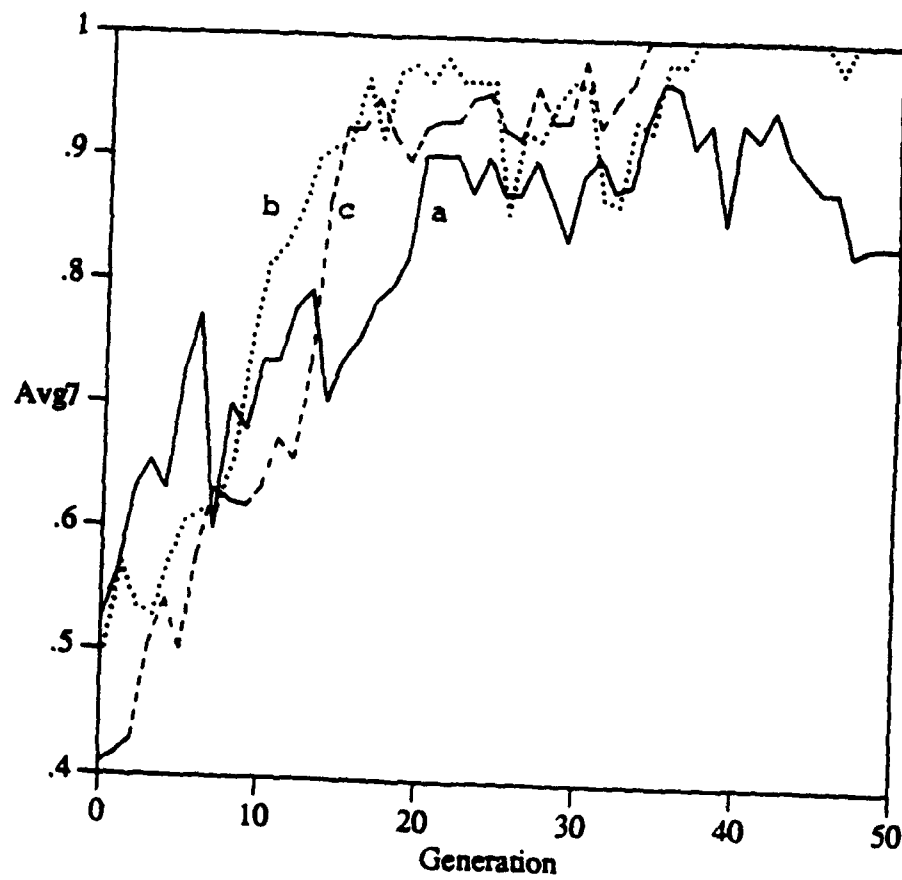
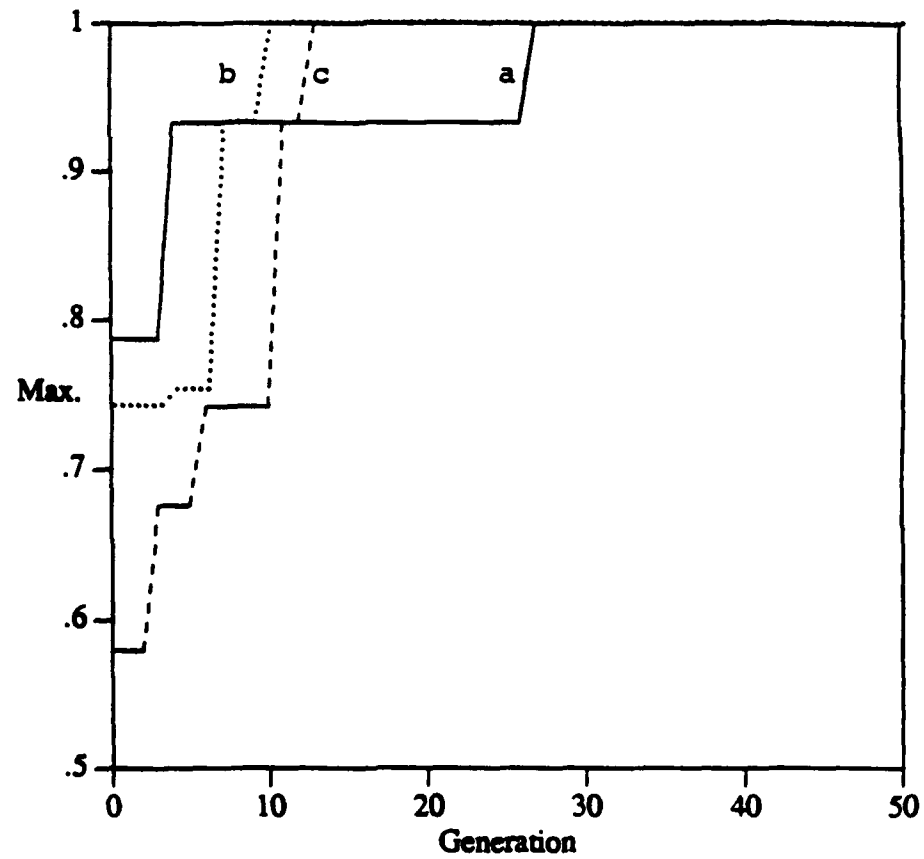
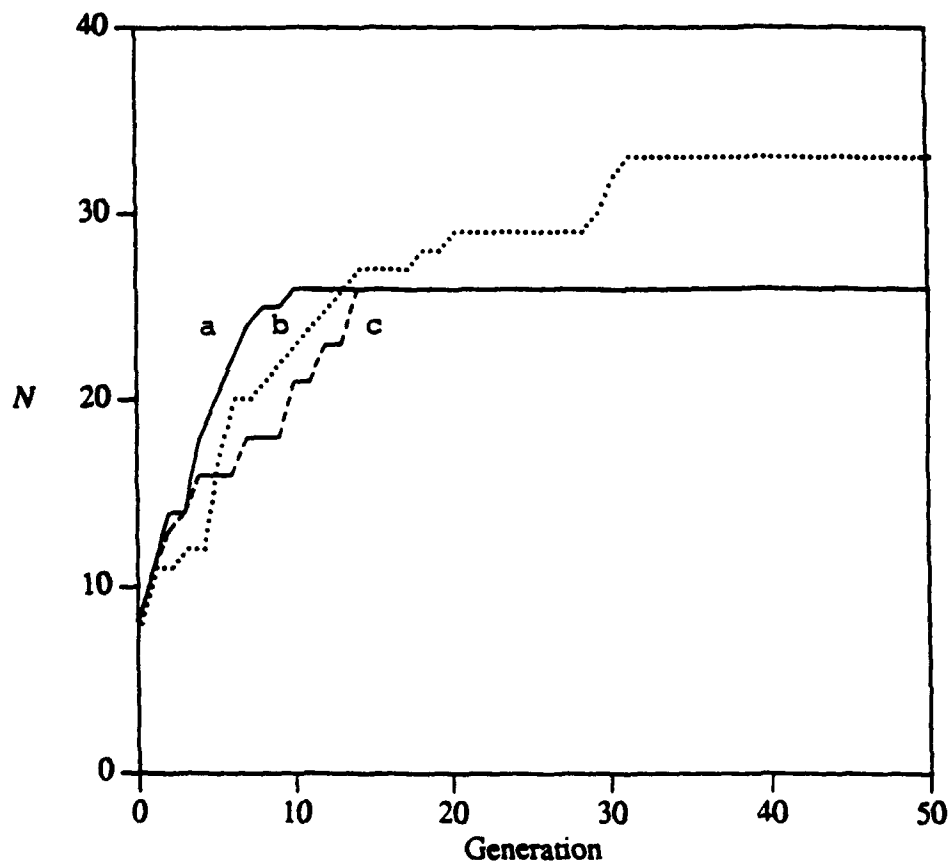


Figure 5.5 Average Fitness Change for 2-Bay Truss with Different I.C.'s  
( $P_c=0.7$ ,  $P_m=0.001$ )

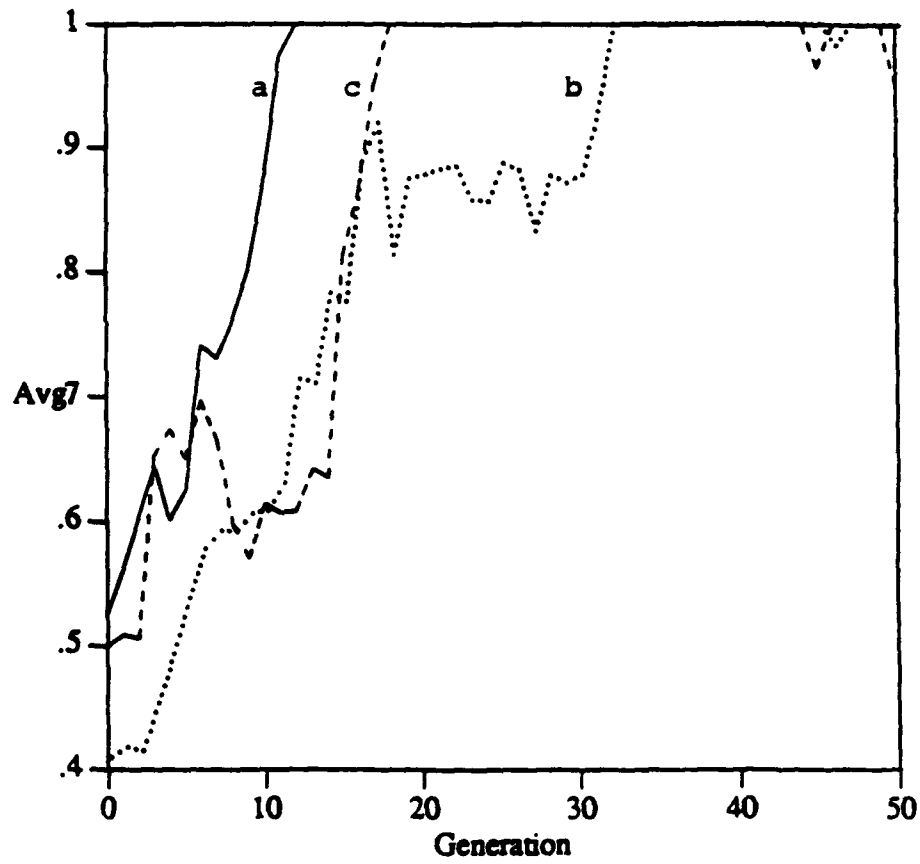


**Figure 5.6** Maximal Fitness Change for 2-Bay Truss with Different I.C.'s ( $P_c=0.7$ ,  $P_m=0.001$ )

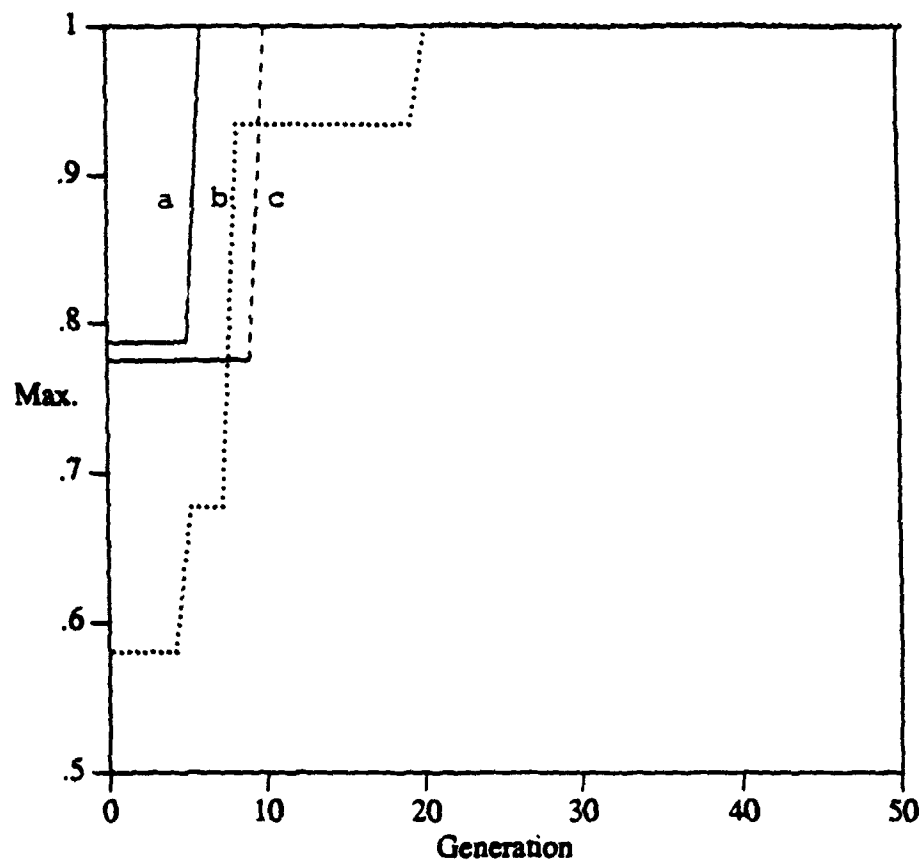




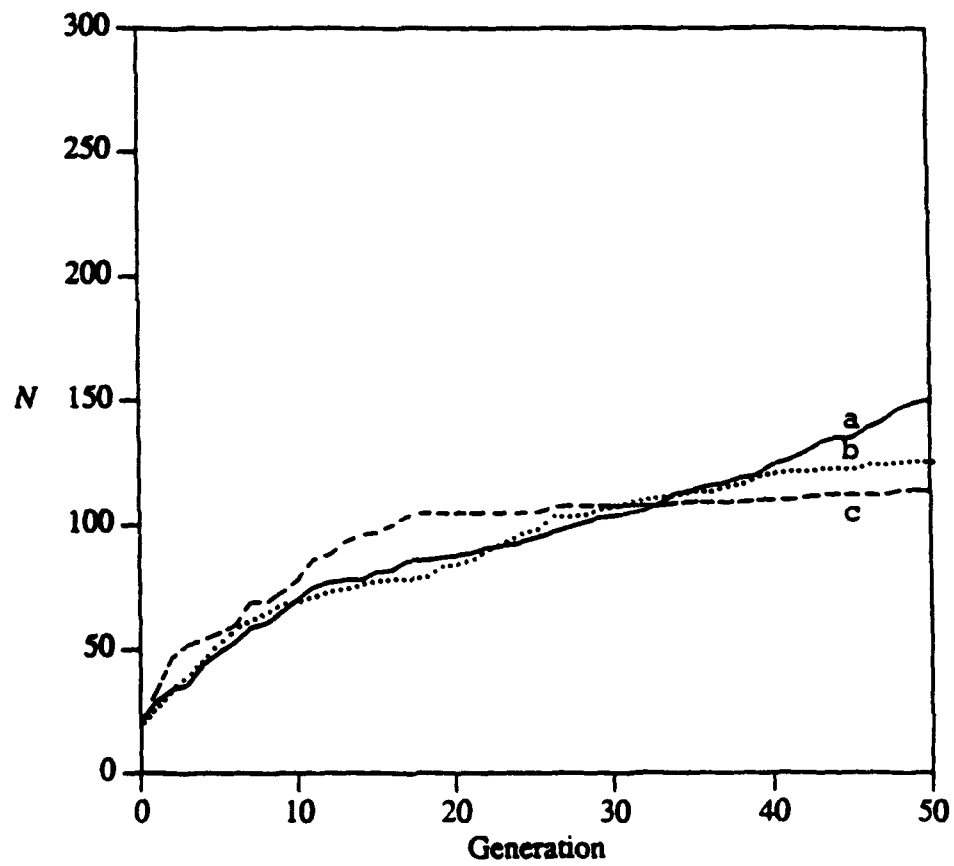
**Figure 5.7** Accumulated Genotypes (Evaluations) for 2-Bay Truss with Different I.C.'s ( $P_c=0.8$ ,  $P_m=0.001$ )



**Figure 5.8** Average Fitness Change for 2-Bay Truss with Different I.C.'s  
( $P_c=0.8$ ,  $P_m=0.001$ )



**Figure 5.9** Maximal Fitness Change for 2-Bay Truss with Different I.C.'s  
( $P_c=0.8$ ,  $P_m=0.001$ )



**Figure 5.10** Accumulated Genotypes (Evaluations) for 6-Bay Truss with Different I.C.'s ( $P_c=0.7$ ,  $P_m=0.001$ )

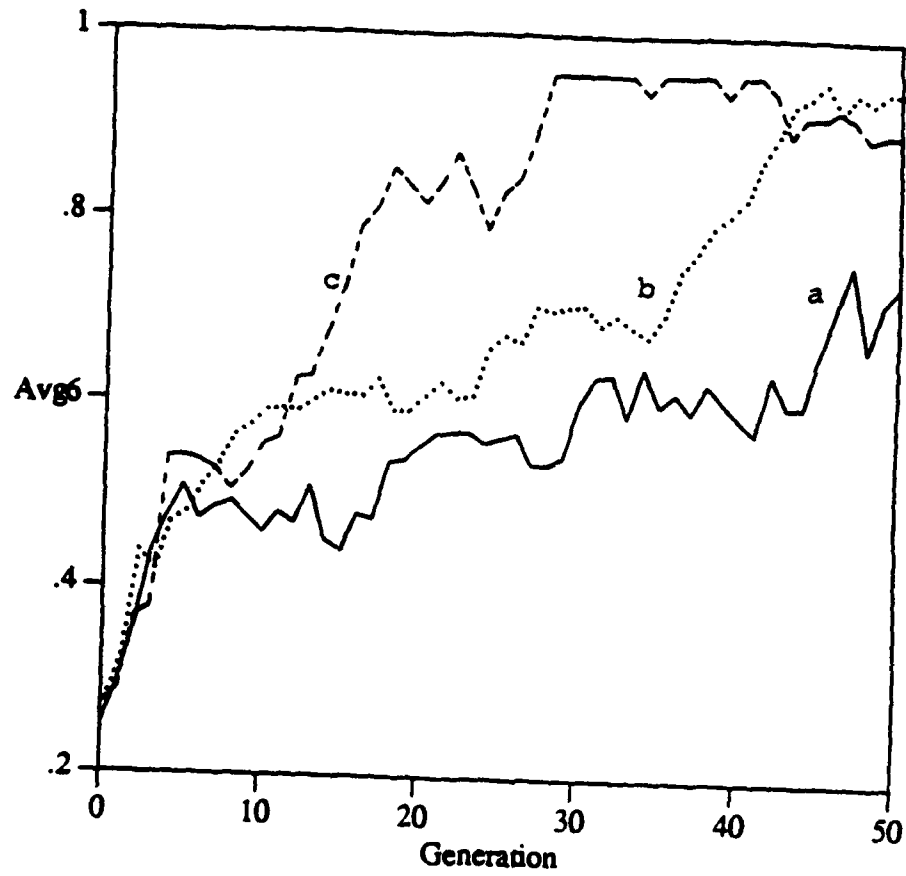
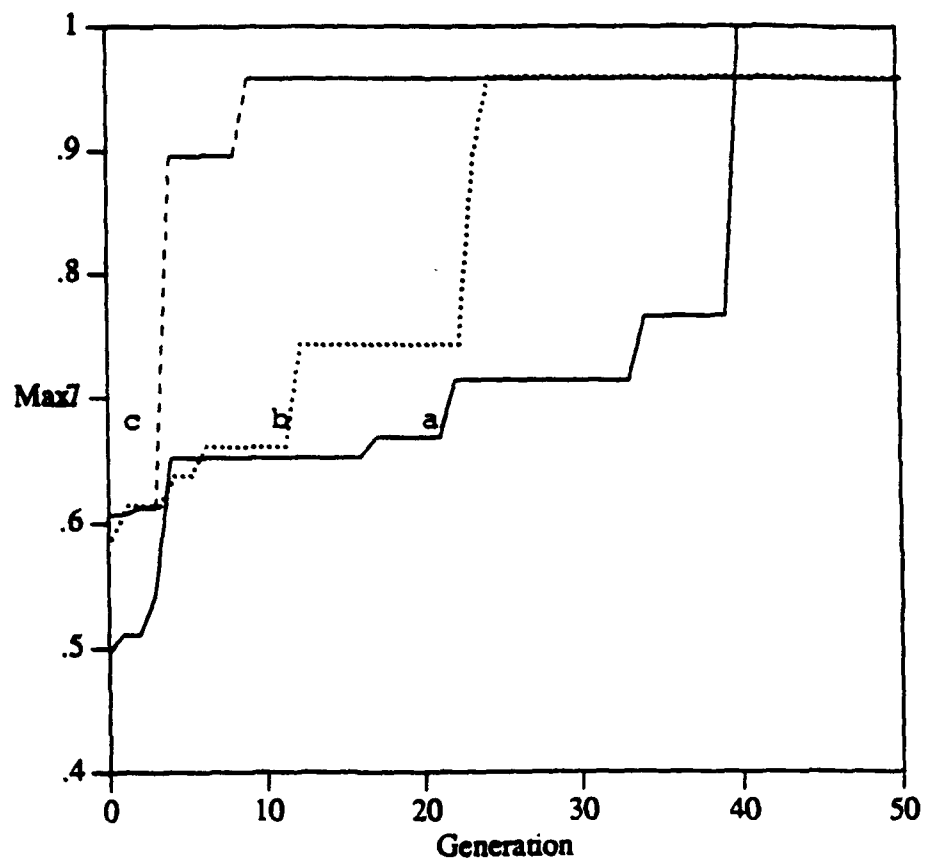
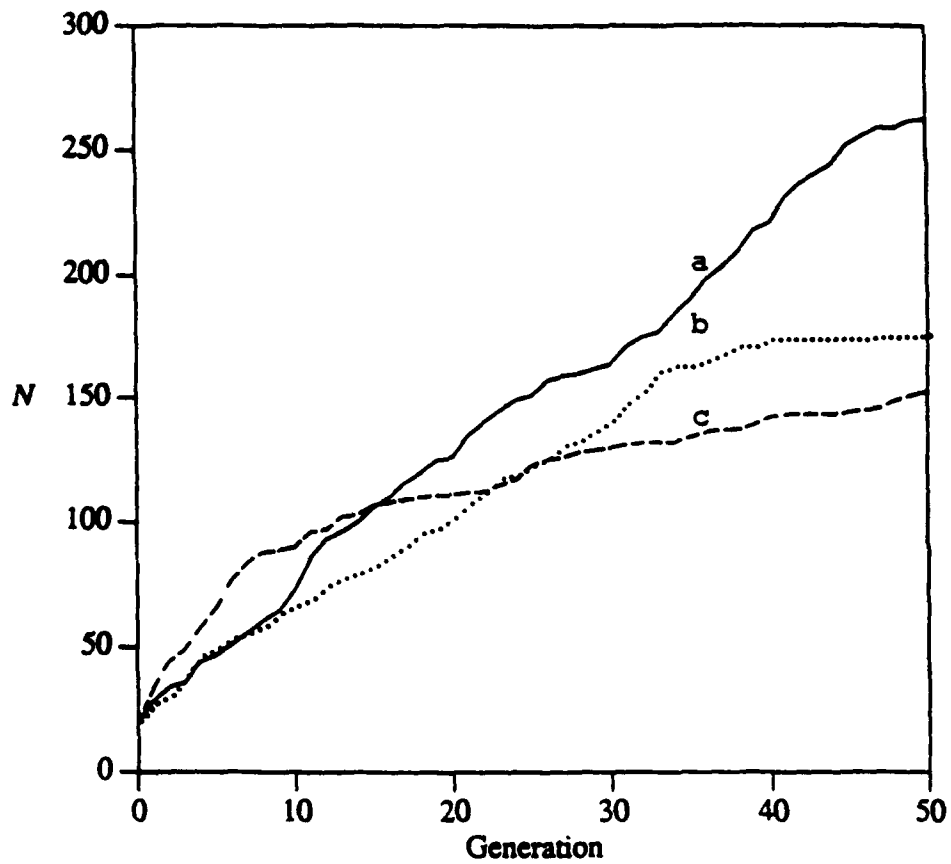


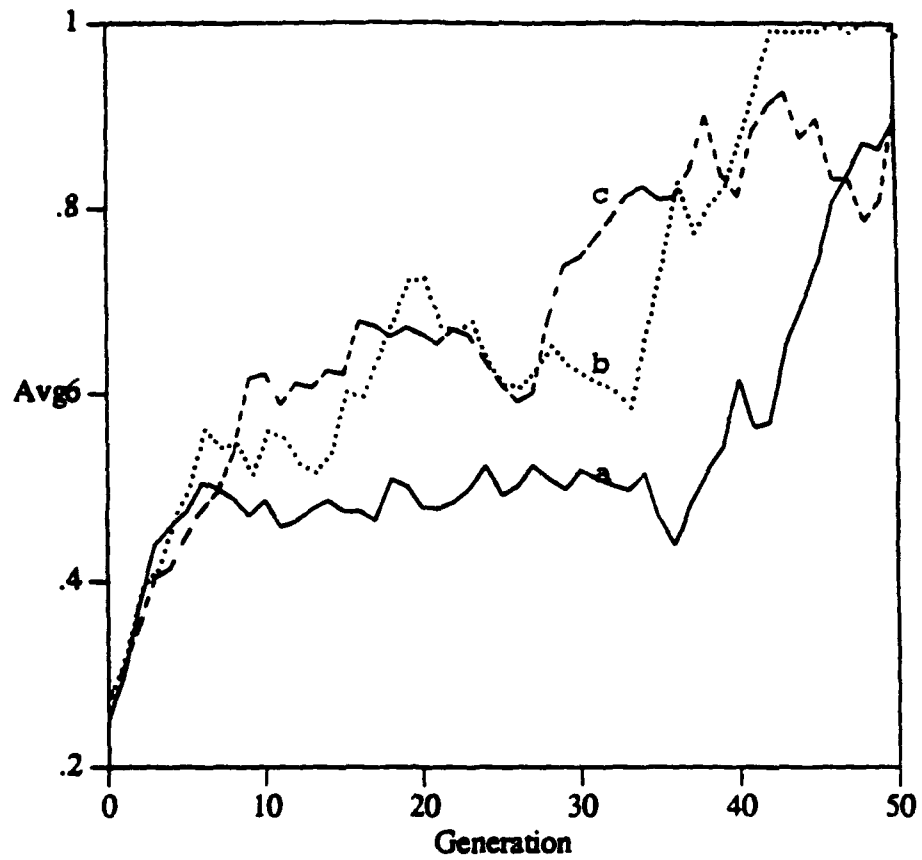
Figure 5.11 Average Fitness Change for 6-Bay Truss with Different I.C.'s ( $P_c=0.7$ ,  $P_m=0.001$ )



**Figure 5.12** Maximal Fitness Change for 6-Bay Truss with Different I.C.'s ( $P_c=0.7$ ,  $P_m=0.001$ )

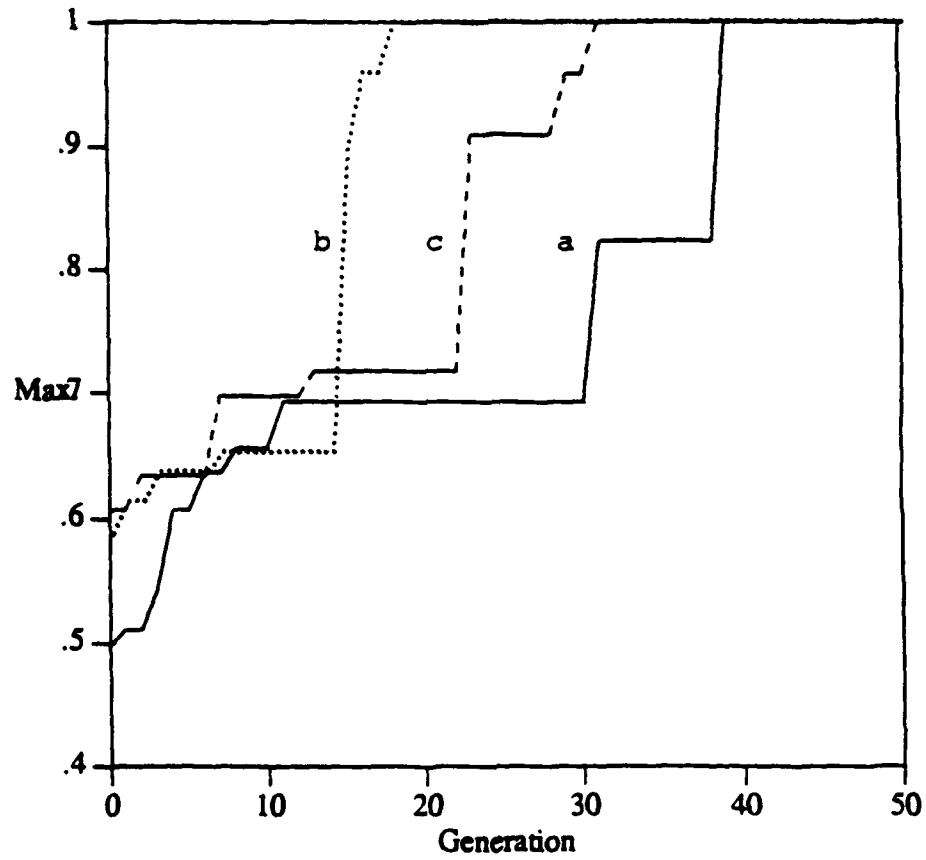


**Figure 5.13** Accumulated Genotypes (Evaluations) for 6-Bay Truss with Different I.C.'s ( $P_c=0.8$ ,  $P_m=0.001$ )



**Figure 5.14** Average Fitness Change for 6-Bay Truss with Different I.C.'s ( $P_c=0.8$ ,  $P_m=0.001$ )





**Figure 5.15** Maximal Fitness Change for 6-Bay Truss with Different I.C.'s ( $P_c=0.8$ ,  $P_m=0.001$ )

## CHAPTER 6

### CONCLUSIONS

#### 6.1 Work in Retrospect

The structural and control design problem of flexible structures is studied. The substructures concept, widely used in large/complex structural analysis, is used to formulate the control system model so that the order of system model for each substructure is small enough for a controller design with no further model reduction needed. Consequently, the interacting substructure decentralized control procedure will give a suboptimal control gain without the spillover problem. The interacting force that is due to the active controller is transmitted through the boundary nodes to the connected substructures and, hence, a modified system model for connected substructures will be made for control design. An iterative procedure is performed for finding the control gain matrix for each substructure. Appropriate assembly of control gains for substructures gives the global control gain. This procedure produces a higher gain compared with that of LQR designed for a complete system model. A unified passive damper design procedure leads to an approximation in the higher order modes without a significant change in the lower order modes. When the dual passive/active control (UPD/ISDC) is used, the system reliability and robustness are increased. The performance index is also reduced by a large amount depending on the quantity of passive damping added.

Since uncertainties are involved in the model for control system design, the robustness of stability and performance should be considered for the closed-loop system. The stability and performance robustness indices are defined and multiobjective optimization techniques are introduced to solve the integrated structural/control design problem. The major motivation of combined structural/control design is that the best control design based on an existing structure might not constitute the best closed-loop system. This complicates the design procedure, but will reduce the number of design iterations and leads to a better design. The utility function, lexicographic and goal programming methods are applied to solve the multiobjective nonlinear programming problem. No unique optimal solution is obtained because of the multimodal nature of the objective function.

The location selection problem for sensors/actuators in large flexible structures is very important and contains a large number of possible combinations for a large system. Several heuristic approaches have been developed by previous researchers. The sequential-best-adding algorithm, penalty function method, and genetic algorithm are introduced in this work. The sequential-best-adding approach reduces the computational effort to a minimum but yields only a local optimum. The penalty function method is not appropriate for this problem. The genetic algorithm seems to be the most promising approach which is demonstrated on simple examples. The genetic algorithm is very simple to implement and a near global optimal

solution is always assured.

## **6.2 Recommendation for Future Research**

(1) Investigate the experimental implementation of interacting substructure decentralized control of large flexible structures.

(2) For integrated structural/control design problem, investigate the feasibility of using the genetic algorithm in multiobjective optimization can be investigated.

(3) More studies on genetic algorithms are needed for application to the sensor/actuator location selection problem of very large structural systems. A parallel processing approach in genetic algorithms is also worthy of investigation.

## CHAPTER 7

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